Design and development of a novel 4-DOF parallel kinematic coordinate measuring machine (CMM)

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1. Introduction

Parallel mechanisms (PMs) consist of a moving platform (MP) and a fixed base, which are connected to each other through at least two kinematic chains. Gough and Whitehall (1962) developed the first hexapod having six kinematic chains in 1954. Many other PMs having various levels of degrees of freedom (DOF) and capabilities were introduced. Although PMs with five or six DOF such as T5 or Gough-Stewart (Song et al. 2014) can be used in the most of industrial applications, the mechanism complexity and cost factors restrict their applications. Clavel (1987) invented Delta robot with three links and three DOF to respond the industry demands such as fast movement of light objects. In addition to this type of mechanism, some four DOF parallel robots, similar to Adept Quattro were developed (Clavel 1987). Recently, PMs have found many industrial applications such as the Coordinate Measuring Machine (CMM). Traditionally, CMMs are design using three linear DOF. In the typical machine tools as well as CMMs with 3 linear DOF, a type of common kinematic error occurs when the tool or stylus axis is not perpendicular to the surface of part (Barari, ElMaraghy and ElMaraghy 2009; Barari, ElMaraghy and Orban 2009; ElMaraghy, Barari, and Knopf 2004; Mahboubkhah, Aliakbari, and Burvill 2016). This orthogonality error causes a severe source of uncertainty in the results of CMM inspection and the larger the stylus tip radius, the greater the measurement error will be. A CMM with four or five DOF has potentially less measurement error than the traditional 3-DOF CMM due to possessing additional rotary DOF. Developing a new CMM mechanism to reduce or eliminate the mutual orthogonality errors between the axes is the main objective in this research.

The new Parallel Kinematics Machine introduced in this paper, namely “C4”, is employed as a coordinate measuring machine. It has four DOF with combination of three rotation and one translation (3T1R). Furthermore, an additional DOF around Z-axis as the fifth DOF of the CMM is provided for an independent rotary table (See Sections 4 and 6). The 5-DOF CMM increases the measuring accuracy by maintaining the stylus perpendicular to the measured surface of the workpiece. The main difference between the proposed mechanisms with the others is in the type of the fourth degree of freedom, which is a rotation about an axis.

In many previous works, such as Adept Quattro mechanism by Clavel (1987), X4 by Xie and Liu (2015, 2016), H4 by Pierrot and Company (1999), I4L by Company, Krut, and Pierrot (2002), I4R by Krut et al. (2004), Par4 by Nabat et al. (2005), and Heli4 by Krut et al. (2006), the fourth DOF is a rotation about the vertical axis of the workspace (Z). Whereas in C4 the fourth DOF is a rotation about the horizontal axis of X. On the other hand, the configurations of the previous mechanisms are very similar to each other, but the C4 mechanism presents an essentially different configuration. The rotation about the horizontal axis (X axis) in PMs Machine tools and CMM allows perpendicularity of the tool or stylus to the surface of the workpiece. This is a useful property for precision machining, or to eliminate the orthogonality measuring error; However, the rotation of tools or stylus around their own axes (Z axis), does not provide a valuable capability.
Liu and Wang (2003) have discussed a preliminary plan of 4-DOF PM with four sliding joints. They noted this mechanism has principle singularity, when the quadrangle of the MP is similar to that of the base. However, they did not introduce a practical solution for this issue.

Eliminating the singularity and increasing dexterity and stiffness are some main design objectives in developing the desired mechanisms. Zhang and Wei (2017) employed the Global Condition Index (GCI) as a criterion to assess the mechanism workspace. They found that increasing the mechanism workspace results in higher mechanism compliance. Many other researchers (Zhang et al. 2012; Wang et al. 2005) have considered GCI as the design criteria of PKM to increase stiffness properties.

In this work, the propounded configuration by Liu and Wang (2003) is investigated as the initial base model in spite of its imperfections. Thereafter, the main challenge is to find a non-singular mechanism with the specified characteristics in its workspace. In addition to formal Jacobian analysis, a novel graphical technique is employed for the singularity analysis in Section 5.1.2. Using this technique, the moving possibility of the MP (singularity or instability of mechanism) can be examined visually using the 3D locus of joints, when the actuators are locked. Furthermore, the validity of the graphical method in each case is evaluated through an experimental test that is performed using a prototype Polyurethane model. This process provides a great visualisation opportunity for a systematic design improvement. The design imprudent is completed systematically by applying rationally decided gradual changes to the fourteen proposed configuration. In this iterative process, the drawbacks for each proposed configuration are examined in details, and their sources are investigated. The results are used to apply the corresponding modifications to the next propose configuration.

The structure of this paper is as follows: In Section 2 the proposed C4 mechanism is introduced and the interconnections between its components are explained in details. In addition, the structural parameters and DOFs of C4 mechanism are introduced in this section. In Section 3, the kinematic equations of mechanism are derived. The workspace of the mechanism is determined in Section 4. Using these relations and the determined workspace, the singularity and dexterity analysis of the proposed mechanism are described in Sec. 5. In addition, the systematic design improvement by iterative study of the design parameters to create a configuration with a well-formed (cylindrical workspace) without any singular point is presented. The stiffness analysis of the mechanism is conducted in Section 6. Section 7 presents the result and discussion, and in Section 8 the conclusion of this research is presented.

2. Introducing the C4 mechanism and its structural parameters

The final configuration of the designed mechanism is illustrated in Figure 1. The mechanism is composed of a fixed base (No. 1) and an MP (No. 7) which are connected to each other via four serial chains (Nos. 2, 3 and 4). The links of A and C are connected to the MP through two revolute joints (Nos. 5 and 6). The rotation capability about X-axis as the fourth degree of freedom is provided to the MP by these joints. Structural diagram of C4 and the relations of its components are illustrated in Figure 2. Sixteen spherical joints and six revolute joints are used in C4 design. The upper parts of chains are connected to the planar four-bar parallelogram. This planar link is a mechanism in which four bars are connected end-to-top in turn by revolute joints. A parallelogram has a higher stiffness with respect to a single bar (Liu and Wang 2003). The rotations of the chains around their own revolute joints are performed by the stepper motors, which are assembled on their revolute joints. Therefore, the
2.1 Mobility of the C4 mechanism

The mobility or mechanism’s DOF is defined as the number of its independent coordinates. The recent investigations on different PMs reported that the classical equations for a quick calculation of mobility have a wrong result in many of cases. Many approaches to calculate DOF are similar to the original or extended forms of the Chebychev-Grübler-Kutzbach (CGK) method (Gogu 2008).

Gogu (2008) has recently proposed a new formula to enable a quick calculation of the mobility of PMs, in which their related parameters can be easily obtained by inspection. This reference provides suitable method to calculate the structural parameters of the complex closed mechanism such as the proposed mechanism in this paper.

- The mobility of the joints (f) and the total mobility of joints, including 16 spherical and 6 revolute joints are as follows:

\[ f = 16 \times 3 + 6 \times 1 = 54 \]  

1

- According to DOF calculations of a similar mechanism to C4 that is conducted in reference (Gogu 2008), the number of joint parameters that lose their independence in the C4 mechanism is:

\[ r = 42 \]  

2

- The mobility of the PM, M is as follows:

\[ M = f - r = 54 - 42 = 12 \]  

3

Indeed, the C4 robot has twelve DOFs, in which according to the following relation, eight of them are known as degrees of redundancy (T).

\[ T = M - S_M = 12 - 4 = 8 \]  

4

where, \( S_M \) is the characteristic connectivity of the mechanism (Gogu 2008).

These eight degrees of structural redundancy represent internal mobility of the limbs that cannot be transmitted from the input to the output of the mechanism. The parallelograms of the mechanism contain two internal mobilities introduced by the rotation of the links 3 and 4 of each limb around a rotation axis passing through the centre of the two spherical joints. These redundancies are restricted in C4 by applying the strip elements in parallelograms, which prevent the rotation of the links 3 and 4 of each limb around their rotation axes. The remaining four external DOF can be transmitted to the output of the robot. Based on this fact, the four moving capabilities of C4 robot including three linear and one angular velocities are illustrated in Figure 1.

3. Kinematics equations

In order to obtain the workspace and investigating the dexterity of C4, inverse kinematic relations of position and velocity are used in the current work. These equations are used to extract the Jacobian matrix and for the dexterity analysis of the mechanism.

3.1 IK of position

The IK of position calculates the angular position of links, when the position and orientation of the MP are given. The relations of C4 components in vector form are illustrated in Figure 3(a) and b. Due to similarity of the kinematic chains in the mechanism, only one of them is shown in Figure 3(a) with ‘i’ index.

The conventional derivation of IK relations due to simplicity have not been considered here. In order to solve the equations, a computer programme is developed. As shown in Figure 1, the reference coordinate frame \( \{O\} \), and the moving coordinate frame \( \{P\} \) are located in the centre of fixed the base and the MP, respectively. As proved in Section 3, the C4 has four DOF, including three linear and a rotation about X-axis, in which the rotation is shown with \( \theta_i \) (See Figure 3(c)).

The rotation angle of \( \mathbf{L}_{2i} \) in \( \{O\} \) (see Figure 3(a)), \( \alpha_i \) as the result of IK solution can be calculated as follows:

\[ \alpha_i = 2\tan^{-1}\left(\frac{-F_i - \sqrt{F_i^2 - (M_i - N_i)(M_i + N_i)}}{(M_i - N_i)}\right) \]  

5

where, in Eq. 5, the scalar parameters \( M_i, N_i \) and \( F_i \) are described as follows:

\[ M_i = [Q_1, Q_2, Q_3, Q_4] + [(-2OB_{x1}a_1), (2OB_{x2} - 2OB_{x3}a_2), (-2OB_{x3}a_3), (-2OB_{x4} - 2OB_{x4}a_4)] \]  

6

where, \( Q_i \) is the position vector of MP in \( \{O\} \) frame. \( \mathbf{L}_{2i} \) and \( \mathbf{L}_{2j} \) are the vectors of lower and upper links, respectively.

\[ \mathbf{OC}_i \) is interconnection point of \( \mathbf{L}_{1i} \) and \( \mathbf{L}_{2i} \)

3.2 IK of velocity

The relative equations between the velocities of the MP and actuators are defined as the kinematic of velocity. These equations are described as IK of velocity, when the velocity of the MP is known and the velocities of actuators are required. IK of velocity can be obtained through a differentiating of the main IK of position equation. The final form of velocity equations in Cartesian and joint coordinate frames \( [\mathbf{X}, \dot{\alpha}] \) are as follows:

\[ \dot{\alpha}_i = \mathbf{J}^{-1} \mathbf{X}, \mathbf{X} = [\dot{p}_x, \dot{p}_y, \dot{p}_z, \omega_{px}]^T \]  

10

where, \( \dot{p}_x, \dot{p}_y, \dot{p}_z, \omega_{px} \) are the linear and angular velocity components of the MP.
where, $J^{-1}$ indicates the principle inverse Jacobian matrix of mechanism as follows:

$$J^{-1} = J_q^{-1}J_x$$

(11)

where, $J_q$ and $J_x$ depict the direct and inverse Jacobian matrices of mechanism, respectively, as follows:

$$J_q = \begin{bmatrix} J_{1y} & 0 & 0 & 0 \\ 0 & J_{2x} & 0 & 0 \\ 0 & 0 & J_{3y} & 0 \\ 0 & 0 & 0 & J_{4x} \end{bmatrix}$$

$$J_x = \begin{bmatrix} L_{11x} & L_{11y} & L_{11z} & b_{1y}L_{11z} - b_{1z}L_{11y} \\ L_{12x} & L_{12y} & L_{12z} & b_{2y}L_{12z} - b_{2z}L_{12y} \\ L_{13x} & L_{13y} & L_{13z} & b_{3y}L_{13z} - b_{3z}L_{13y} \\ L_{14x} & L_{14y} & L_{14z} & b_{4y}L_{14z} - b_{4z}L_{14y} \end{bmatrix}$$

(12)

where, $[J_{ix}, J_{iy}, J_{iz}]^T$ is as follows:

$$\begin{bmatrix} J_{ix} \\ J_{iy} \\ J_{iz} \end{bmatrix} = \begin{bmatrix} L_{2y}L_{11z} - L_{2z}L_{11y} \\ L_{2z}L_{11x} - L_{2x}L_{11z} \\ L_{2x}L_{11y} - L_{11x}L_{2z} \end{bmatrix}$$

(13)

### 3.3. Direct kinematic of position

Direct kinematic problem, calculates the Cartesian position and orientation of the MP, when the angular positions of links are known. In the proposed CMM robot, the position and orientation of probe in its trajectory are required. For this purpose, the direct kinematic programme, which was written in C++ environment for controller of mechanism, is employed to calculate the pose and orientation of the CMM probe head. The angular positions of links as the input data of direct kinematic problem are provided by the rotary encoders, which are coupled with motor shafts (Section 6).

Various methods have been presented to find the solutions of the direct kinematic equations. However, some of which need longer computing time and result in meaningless responses. Among these solutions, the numerical methods using a-priori information, such as the Newton-Raphson method (NR) or Jacobian free monotonic descent method (JFMD) (Shen et al. 2016) can be applied to solve direct kinematic of the robot. In this paper, both of these methods were used to solve direct kinematic equations of the developed mechanism. In NR method the initial guess, which is needed to begin the iteration, should be located in the domain of convergence; otherwise convergence may not be occurred. In JFMD method, the approximated Jacobian matrix is added to the modified NR iteration, to conquer the divergence of the algorithm with poor initial guess. Therefore, in current research, JFMD method with the highest performance and considering the real-time necessity for solving the direct kinematic equations has been used.

### 3.4. Validity inspection of the kinematic relations

In order to investigate the validity of the kinematic equations and their related programmes, a 3D model simulation of the
The proposed robot was constructed in a commercial Computer Aided Design (CAD) environment. Using this simulation, the end-effector of the robot is located in some predefined positions and orientations and then the related angular positions of links are measured accurately. This simulation establishes several inputs and outputs for verifying the kinematic problems. For instance, according to Figure 3(a and b) (one of the developed configurations in CAD) the rotation angle of \( L_2 \) and the position and orientation of MP in \( O \) can be obtained using measuring tool in CAD. Using this data, the inverse and direct kinematic programmes are solved. The direct kinematic programme is solved using the angular position of links with an initial guess for end-effector position. Finally, the comparison of the results proves that the answers of the inverse kinematic solution correspond to the CAD simulation.

4. Workspace of mechanism

In order to design the workspace of C4, both geometrical and discrete methods are conducted. An exploring algorithm with an appropriate resolution scans the prescribed region to find the boundary of precise workspace. At last, the selected points are checked considering constraints and limits of the mechanism and are accepted as the part of workspace when passing the prescribed conditions (Stamper, Tsai, and Walsh 1997). To this end, all of the kinematics and geometrical constraints as well as the collision possibility of the links are inspected. It is desired to have the shape of the workspace preferably symmetric and regular (close to the cylindrical form that is located on the table of CMM as illustrated in Figure 4). This is referred here as a well-formed workspace. Several variations of the design parameters should be considered to achieve a well-formed workspace. These variations consist of available sizes and shapes of mechanism’s components as well as the possible states of their connections.

Therefore, in order to specify the final form of the mechanism, a study on selecting the effective design parameters as well as determining the size limits of the components (the dimensions of \( L_{1}, L_{2}, a, \) and \( b_{1} \) ) is conducted.

5. Design evaluation criteria of the mechanism

In order to evaluate the proposed mechanisms, some main design evaluation criteria are considered in the current work. The most important evaluation criterion is the non-singularity of the mechanism in its workspace. In addition to the singularity, dexterity and stiffness quality of the workspace points in a neighbourhood of the singular points have also key importance on its performance.

A PM with poor design and imperfect dexterity exhibits a strange behaviour whenever the revolute actuators of mechanism are locked, the end effector moves severely. Gosselin (1985), Zlatanov, Bonev, and Gosselin (2003), Merlet and Daney (2005) and Akhbari, Ghadimzadeh Alamdari, and Mahboubkhhah (2019) reported this behaviour for the non-dexterous robots. Many researchers have referenced this status by analysing and investigating the Jacobian matrix of mechanism. Hence, the singularity, dexterity and quality of the C4 workspace are analysed in this paper.

5.1. Singularity analysis of mechanism

Since the parallel robot is unstable at singular points, it is preferred to avoid the singular configurations in design iterations. Three methods, an analytical, a graphical and an experimental method are employed in this research to evaluate singularity as well as the near singularity conditions.

5.1.1 Analytical method

The singularity in a parallel robot happens when the determinant of the inverse Jacobian matrix is equal to zero. According to Eq. 26 and in the calculation process of \( X \), the direct singularity happens when the determinant of \( J_{x} \) is zero \((|J_{x}| = 0)\). Besides, according to Eq. 26 in obtaining the \( q \), the pose of the workspace is inversely singular when the determinant of \( J_{q} \) is zero \((|J_{q}| = 0)\). In this condition, at least one of the diametric arrays of \( J_{q} \) is zero.

In design iterations of C4, the configurations with both of the direct and inverse singularities are refused and an appropriate configuration without singular points has been searched by a systematic direct search approach (Section 5.3).

5.1.2 The novel graphical method

The developed graphical method in this work provides the 2D and 3D loci of MP centre with the other common revolute joints. The loci of revolute joints, which are extracted in this method, enable evaluating the displaced possibility of the MP, whenever the revolute actuators of mechanism are locked. Furthermore, this method allows evaluating the reason of singularity and instability of the proposed mechanism during the design process. For instance, the intersecting of the MP centre with revolute joints of
the links No.2 and No.4 is extracted through their 3D loci. Besides, the common 3D locus of the revolute joints of links No.1 and No.3 is obtained using CAD simulation (Section 5.3). Then, the common 2D locus of the MP centre and revolute joints of the links No.1 and No.3 is extracted using the previously obtained 3D loci. Finally, the direction and distance between the resultant loci are used to interpret the singularity condition of C4. In this case, when the mechanism is in its singular condition and motors are off, the MP in company with the revolute joints can move along two intersecting ways. In the next sections, various configurations are analysed using this technique.

5.1.3 Experimental method
The analytical method for singularity and dexterity analysis of the mechanism is neither clear nor efficient. In addition, there are usually some inconsistencies in the solutions resulting by the analytical model due to simplifying assumptions. The detailed complexity of configuration and loading system acting on the structure are often neglected in analytical modelling and it is difficult to develop exact mathematical solution for their design. As a result, in this research, the simple models of the specified configurations were constructed from the Polyurethane material. The permanent joints of the components were constructed from bolts and glue. The instability, singularity and weak dexterity conditions of the model were evaluated through loading the MP and investigating its displacement (Section 5.3).

5.2. Dexterity analysis of mechanism
In order to indicate the dexterity of workspace points, the homogenous Jacobian matrix of mechanism (Jh) is decomposed to three matrices using Singular value Decomposition (SVD) technique (Stamper, Tsai, and Walsh 1997; Gohari and Barari 2016). The condition number that is obtained using aforementioned technique is applied as an index to explain accuracy, dexterity and closeness of a position to a singular point.

The inverse of condition number, which has a value between 0 and 1 as the Local Dexterity Index (LDI), describes the overall kinematic behaviour of mechanism.

A value of zero for LDI indicates that Jacobian matrix is singular. Whenever the LDI in a described pose of a workspace is equal to unity, the mechanism will be in its ideal condition from the singularity, dexterity and stiffness viewpoints. In this state, the velocity and movements of the links have appropriate relations. In order to evaluate the dexterity of a mechanism over the total workspace, Gosselin (1985) and Kong and Gosselin (2007) have introduced the Global Conditioning Index (GCI).

The Global Dexterity Index (GDI) shows the mean value of the LDI and explains the optimum design quality of a mechanism in its overall workspace. In the next section, in addition to calculating the GDI value for the different configurations of the mechanism, the LDI values for all points of the workspace are also obtained. Then the workspace of each proposed mechanism is classified based on their corresponding LDI values.

5.3. Systematic design improvement
The design evaluation criteria require the proposed mechanism to be non-singular in its workspace, the points of workspace to have the highest LDI value, the number of points with high LDI value to be maximised in the workspace, and the general GDI value of the selected configuration to be in its highest possible value. In order to show the trend of the systematic design improvement for the C4 mechanism, the gradual changes in the fourteen proposed configurations are discussed in this section. As can be seen in Table 1, the dimensions of the links L1 and L2 and principle variations of the fixed and MPs a and b, are investigated as the effective design parameters (Figure 3(a and b)).

The first model is selected based on the proposed configuration by Liu and Wang (2003). In this case, the total form and the components of the four links are symmetric (Figure 5(a)). So that, even though the upper links of mechanism are locked, by applying a small external force to the MP, it moves along YZ-plane. This fact is revealed by an experimental test that is performed on a Polyurethane model as can be seen in Figure 5(b). In this case, the MP by applying F, moves along the intersecting paths of MP joints and, consequently, the mechanism would be unstable in this plane (Figure 5(b)). In Figure 6(a), the common locus of the MP centre and the links No.2 and No.4 as well as the common locus of revolute joints of the links No.1 and No.3 are illustrated. In this case, due to parallelism of both paths, all four mutually joints of the links in company with the MP can move along this path. Figure 6(b) shows the 3D locus of the MP joints. The 2D locus of the MP and its revolute joints in Figure 6(c) depicts that the MP can move along two intersecting lines when the motors are off due to a fixed distance of the revolute joints. Thus, the mechanism is in its singular condition. The singularity of mechanism No.1 in YZ-plane which is recognised by Jacobian matrix analysis is illustrated in Figure 7. The GCI value of this configuration is relatively large. Moreover, the workspace has symmetric form due to symmetric model of the MP. These investigations reveal that the model No.1 is not impractical.

The configurations of models No.2, 3 and 4 are illustrated in Figure 8, Figure 9 and Figure 10, respectively. All of the configurations of Table 1 were investigated using similar methods mentioned above with the aims to increase the height and radius of the workspace, non-singular conditions and the highest dexterity in the workspace.

The model No. 4 (Figure 10) has not singularity in its workspace. According to Figure 11, the intersecting locus of the

<table>
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<th>No. of Config.</th>
<th>a</th>
<th>b</th>
<th>L1 = 1,3</th>
<th>L2 = 1,3</th>
<th>θ</th>
<th>MP Config.</th>
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Figure 5. First model of mechanism (No. 1). (a) Proposed symmetrical plan of MP, (b) Polyurethane model.

Figure 6. Graphical representation of paths of joints on MP related to model No. 1. (a) Locus of joints on MP, (b–d) locus of joints on MP, (c). Intersection path of joints on MP.
According to this outcome, it reveals that the position of the joints on the MP has a very critical role and it should be asymmetric as much as possible. This guideline was observed in the next configurations.

In the configuration No.12, the LDI value of many workspace points are in the highest range (0.01–0.1) and the GCI of this configuration has also the highest value. Besides, the shape of workspace is in a desired cylindrical form that is located on the CMM table (Figure 4).

Since the orientations of the MP changes the form of the workspace, the orientation effect of the MP was also investigated in the middle of the design process. For instance, configurations 13 and 14 depict the workspace of the mechanism with inclined states of the MP. In these states, the form of workspace will be relatively asymmetric. The general results of these investigations are depicted in Table 2.

6. Stiffness analysis of the C4 mechanism

The evaluation of force relations and stiffness design of the PMs is one of the important design criteria, which should be considered in routine manufacturing process. Some researchers (Svinin, Hosoe, and Uchiyama 2001; Company, Krut, and Pierrrot 2002) have considered stiffness analysis and elastic performance evaluation of PMs. Similarly, the displacement of C4 mechanism under external forces applied on its MP has been evaluated in this research, using both the Finite Element Method (FEM) and analytical method. Since, this is a straightforward and lacks novelty, it is not detailed in the paper. The dimensions of the components (for instance, the diameters of rods) concerning the required strength of materials are determined and designed through these analyses.

The optimum configuration based on the analyses results presented in this paper, was manufactured, as the final design (Figure 12).

In addition, the experimental test setup for exploring the possible small displacement of the MP under applied forces to the MP is conducted (Figure 13). To this end, the motors and the upper links of the mechanism are locked; simultaneously, the defined load is applied to the MP in X, Y and Z directions, respectively, and the related displacements are measured using a dial indicator. This test is performed in several points of the workspace having different LDI values. Although the C4 mechanism is designed as a CMM and its MP carries mutual joints of the links No.2 and No.4 with the revolute joints of the links No.1 and No.3 are not parallel; therefore, the MP can’t be moved and this model is continuously non-singular.
a scanning probe, the probing load on the MP is less than (1N). Therefore, in experimental tests, maximum load applied to the end-effector is about (1N). Under this load, the displacement of the MP is measured as presented in Table 3.

7. Result and discussion

The tradition presented PKM in the current paper is completed by trial the primary plan proposed by Liu and Wang (2003). However, the integration of the analytical approach, innovative graphical method, and the experimental study to find the non-singular and dexterous PM as presented in Table 1 resulted in a systematic design approach in this work. The graphical representation of the paths of the joints in each step was used to diagnose and modify the singularity of the mechanism. Furthermore, the real efficiency of the proposed configurations was evaluated through experimental tests performed on the manufactured models. At four first configurations, presented to Table 1, the mentioned trend was chosen to find non-singular mechanism in its workspace.

In order to find the best dexterity of the mechanism, and to define the numerous parameters including available sizes of the mechanism’s components and the possible states of their connections with each other, the following proposed configurations (configurations No. 5 to No. 14) were considered. In this base, the drawback of each proposed concept was revealed and in the

<table>
<thead>
<tr>
<th>No. of Config.</th>
<th>Status of Workspace</th>
<th>GCI</th>
<th>Min. Diameter× Min Height (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Singular</td>
<td>0.023</td>
<td>240 × 210</td>
</tr>
<tr>
<td>2</td>
<td>Singular</td>
<td>0.028</td>
<td>250 × 225</td>
</tr>
<tr>
<td>3</td>
<td>Singular</td>
<td>0.024</td>
<td>260 × 225</td>
</tr>
<tr>
<td>4</td>
<td>Nonsingular-Asymmetric in YZ-plane</td>
<td>0.023</td>
<td>250 × 220</td>
</tr>
<tr>
<td>5</td>
<td>Nonsingular-Asymmetric in YZ-plane</td>
<td>0.012</td>
<td>320 × 275</td>
</tr>
<tr>
<td>6</td>
<td>Nonsingular-Partially Symmetric</td>
<td>0.015</td>
<td>310 × 270</td>
</tr>
<tr>
<td>7</td>
<td>Nonsingular-Partially Symmetric</td>
<td>0.016</td>
<td>320 × 275</td>
</tr>
<tr>
<td>8</td>
<td>Nonsingular-Symmetric</td>
<td>0.015</td>
<td>260 × 250</td>
</tr>
<tr>
<td>9</td>
<td>Nonsingular-Symmetric</td>
<td>0.020</td>
<td>320 × 275</td>
</tr>
<tr>
<td>10</td>
<td>Nonsingular-Symmetric</td>
<td>0.020</td>
<td>250 × 300</td>
</tr>
<tr>
<td>11</td>
<td>Nonsingular-Partially Symmetric</td>
<td>0.017</td>
<td>260 × 250</td>
</tr>
<tr>
<td>12</td>
<td>Nonsingular-Symmetric</td>
<td>0.020</td>
<td>320 × 300</td>
</tr>
<tr>
<td>13</td>
<td>Nonsingular-Partially Symmetric</td>
<td>0.018</td>
<td>250 × 280</td>
</tr>
<tr>
<td>14</td>
<td>Nonsingular-Partially Symmetric</td>
<td>0.018</td>
<td>250 × 280</td>
</tr>
</tbody>
</table>

Figure 11. Intersecting path of joints on MP related to model No. 4.

Figure 12. The manufactured C4 mechanism.
Figure 13. Experimental displacement test of C4 mechanism.

Table 3. Displacement of end-effector under applied force (1N) in four different locations of the workspace with different LDI values related to model No. 12.

<table>
<thead>
<tr>
<th>No. of configuration</th>
<th>Quantity of LDI</th>
<th>Displacements of MP(µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10^{-4} &gt; \text{LDI} &lt; 10^{-3}$</td>
<td>$ΔX$ $ΔY$ $ΔZ$</td>
</tr>
<tr>
<td>2</td>
<td>$10^{-3} &gt; \text{LDI} &lt; 10^{-4}$</td>
<td>5 7 4</td>
</tr>
<tr>
<td>3</td>
<td>$10^{-2} &gt; \text{LDI} &lt; 10^{-3}$</td>
<td>5 5 3</td>
</tr>
<tr>
<td>4</td>
<td>$10^{-1} &gt; \text{LDI} &gt; 10^{-2}$</td>
<td>2 3 2</td>
</tr>
</tbody>
</table>

next iteration, the required modifications to reach the suitable quality of LDI distribution and covering a cylindrical shape of the workspace were depicted (Table 2). Finally, it is found that, in the configuration No.12 in Table 1, the most of LDI value of the workspace points are in the highest range (0.01–0.1) and the GCI of this configuration has the highest value. Besides, the workspace shape is in the desired cylindrical form that is located on the CMM table (Figure 4). Then, the dimensions of the components concerning the required strength of the materials are determined through the FEM and an analytical method. Hereby, the final configuration was constructed and the experimental tests on the manufactured mechanism according to setup in Figure 13 to achieve the maximum displacement of the MP due to exertion of a probing load. It is evident from Table 3 that the displacement of the MP is smaller in the workspace with large LDI value than the workspace with small LDI value. In addition, the experimental test depicts that the final designed mechanism is completely stable even at the weakest poses of the workspace with an acceptable displacement. According to Figure 12 the developed PM's motion resolution depends on resolution of step motors, the ratio of used gearboxes and kinematics of the system. The kinematics of the structure discussed previously in Section 3. The micro step per revolution of employed motor is 40,000. Furthermore, as the ratio of used zero-backlash gearboxes are 10:1, they increase the output torque and the rotational resolution 10 times. Taking all into account, the driving system is able to generate a rotational resolution of 0.0009° for each of the arms which lead to a linear motion resolution of about 3 µm through the workspace.

8. Conclusion

Despite the fact that the general six DOF PMs are used in the most of the robotic applications; The mechanisms with fewer DOF have found more industrial demands considering some of their advantages such as lower cost, less energy usage and easier control pattern as well as faster movement. In the current paper, a new parallel robot with 4-DOF was introduced that is employed as a CMM base. The final configuration of the MP has C shape with four links connected to it and accordingly the mechanism is named C4. The main difference between the C4 with the other similar mechanisms is in the type of the fourth DOF, which is a rotation around an axis. In order to achieve the design and investigate the performance of the proposed mechanism, systematic analyses were conducted. The inverse and direct kinematic equations of mechanism were obtained and an exploring algorithm to find the workspace of the mechanism was employed. In addition to the analytical singularity analysis, novel graphical and experimental methods to detect singularity behaviour of the MP were applied. By integrating the analytical approach, the developed graphical method, and the experimental method, a systematic design improvement process was completed to consider the effect of possible different variations, such as the dimensions and the interconnection of the components. The design evaluation criteria and the efficient parameters of the mechanism were defined to conduct this process. Several concepts of the system were investigated and the mechanism with the highest dexterity and stiffness, which has a symmetric and practical workspace (without any singular points), was proposed. The selected concept was manufactured and the experimental tests for exploring the quality of the mechanism were performed. The investigations in this research depict that the structure of the C4 robot due to its acceptable design concerning non-singularity, dexterity and strength of the materials conditions, maximally has about seven micrometre displacements under external loads. However, this common error can be compensated through the further calibration process to improve the final accuracy of CMM.

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