Magneto shot noise in noncollinear diffusive spin valves

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We develop a semiclassical Boltzmann-Langevin theory for the spin polarized shot noise in a diffusive normal metal spinvalve connected by tunnel contacts to ferromagnetic reservoirs with noncollinear magnetizations. We obtain basic equations for correlations of the fluctuating spin-charge distribution and current density matrices by taking into account the spin-flip processes and spatial precession of the spin accumulation vector in the normal metal. Applying the developed theory to a two terminal ferromagnet-normal metal-ferromagnet (FNF) structure, we find that for a small spin-flip strength and a substantial polarization of the tunnel contacts the shot noise has a nonmonotonic variation with the angle between the magnetization vectors. While the shot noise is almost unchanged from the normal structure value for parallel configuration and increases well above the normal value for an antiparallel configuration, it suppresses substantially at an intermediate angle depending on the ratio of the conductances of the N metal and the tunnel contacts. We also demonstrate the pronounced effects of the polarization and the spin-flip scattering on the shot noise which reveal the interplay between relaxation and precession of the spin accumulation vector in the N metal.

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I. INTRODUCTION

Spin-polarized transport in magnetoelectronic structures has recently attracted an intense interest largely due to its important technological applications such as nonvolatile magnetic random access memories (MRAMs), read heads for mass data storage, and highly sensitive magnetic sensors.1-3 The main effect is the (giant) magnetoresistance4,5 in magnetic multilayers and spinvalves, i.e., the remarkable decrease in the resistance when the orientation of the magnetization vectors of the ferromagnetic regions changes from antiparallel to parallel. A spinvalve consists of two ferromagnetic leads as the spin injector and detector, connected through a normal metal spacer which serves for the spin accumulation. Recent interest in transport with noncollinear (neither parallel nor antiparallel) magnetizations was stimulated by the spin-transfer magnetization torque,6,7 which is essential for the new devices such as the spin-flip8,9 and spin-torque transistors.10

The most appropriate theoretical formalism for noncollinear ferromagnet-normal metal structures is the magnetoelectronic circuit theory.5 This theory is based on dividing the structure into the reservoirs, nodes, and the junctions and expressing the currents in terms of the scattering matrices of the junctions and isotropic electronic distributions inside the nodes and reservoirs.11 For the noncollinear spin-transport the spin-charge currents and corresponding distributions inside the nodes and reservoirs, as well as the scattering matrices acquire the $2 \times 2$ matrix characteristics of the Pauli spin space. The semiclassical Boltzmann equation provides a standard method to calculate the noncollinear distribution matrix in a continuous bulk medium in the diffusive limit. Magnetoelectronic circuit theory complemented by the Boltzmann diffusion equation has been widely used to study different magnetoelectronic dc effects.12 In spite of this there has been little attention paid toward fluctuations of the noncollinear spin-polarized current. Time-dependent current fluctuations at low temperatures, the so-called shot noise, provide valuable information about the transport process which are not extractable from the average current.13,14 In magnetoelectronic structures, in which the spin of electrons plays an essential role, the shot noise is expected to contain spin-resolved information, including spin-dependent correlations and spin accumulation and relaxation. This together with the importance of the noise in magnetoelectronic devices in view of applications motivates studying of the spin-polarized current fluctuations.

A theory for current fluctuations in disorder conducting systems can be formulated by including the Langevin sources of fluctuations due to the random scattering from disorders and the fluctuations of the distribution function in the semiclassical Boltzmann equation.16 The resulting Boltzmann-Langevin (BL) formalism has been used to calculate the shot noise in many mesoscopic structures and its results are widely believed to be identical with an ensemble average of the exact quantum results.13 One important example is the universal one-third suppression of the shot noise in a diffusive normal metal,15,16 which was correctly predicted within the BL approach.17,18 Recently spin-polarized shot noise has been studied theoretically in a normal metal connected to the ferromagnetic terminals with collinear magnetizations.19-22 In Ref. 21 a semiclassical BL theory of the collinear spin-polarized current fluctuations in a diffusive normal metal was developed. It was found that in a multiterminal spin-valve structure the shot noise and the cross correlations measured between fluctuations of the currents of two different ferromagnetic terminals can deviate substantially from the unpolarized values, depending on the spin accumulation and spin-flip scattering strength. On the other hand only a few works23,24 have been devoted to shot noise in noncollinear systems. Very recently Braun et al.,23 showed that the frequency-dependent shot noise in a quantum dot spinvalve with noncollinear magnetizations of ferromagnets in the presence of an external magnetic field can be used to detect single-spin dynamics in the quantum dot.

Tserkovnyak and Brataas24 extended the circuit theory to obtain the Landauer-Buttiker (LB) formula for the shot noise
of a FN contact with a spin current noncollinear to the magnetization of F. They analyzed the angular dependence of the shot noise in a N metal node connected to two F reservoirs for different types of junctions between the reservoirs and the node. However, the effect of the spin-flip scattering inside the N metal as well as the spatial variation of the noncollinear spin accumulation vector, which are essential in diffusive spin valves, have been disregarded so far. Including these effects requires an extension of the BL method to contain the diffusion and relaxation of the noncollinear spin accumulation inside the diffusive N metal. To our knowledge there is no BL theory for fluctuations of a noncollinear spin-polarized transport. The aim of the present work is to develop such a theory.

In this paper we develop semiclassical BL equations for the noncollinear spin-polarized current fluctuations in the presence of the spin-flip scattering. We obtain the basic diffusion equations for the fluctuating spin-charge distribution and current matrices. These equations allow us to calculate the mean current and the correlations of the corresponding fluctuations in a diffusive normal metal connected by tunnel contacts to several F reservoirs. The developed BL equations are supplemented by the generalized LB formula for the shot noise at the contact points. To illustrate the main behavior of the shot noise in noncollinear magnetoelectronic systems we apply the developed BL equations to calculate the Fano factor, defined as the ratio of the noise power to average current, in a two terminal FNF structure. The noncollinear orientation of the magnetizations causes a spatial precession of the spin accumulation vector (rotation of the spin polarization direction) through the N metal. In the presence of the spin-flip scattering it is associated with a damping as a function of the distance from the F reservoirs. For a small spin-flip intensity and a finite polarization of the tunnel contacts the precession of the spin accumulation results in a nonmonotonic angular dependence of the Fano factor. For a parallel configuration the Fano factor is almost the same as the normal state value but for an antiparallel configuration it increases well above this value due to a large spin accumulation in the N metal. We find a substantial decrease of the Fano factor at an intermediate orientation determined from the conductances of the N metal and the tunnel contacts. Introducing the spin-flip scattering as well as decreasing the spin polarization diminish the nonmonotonic behavior. We present a full analysis of the magneto shot noise which demonstrates the effects of the spin-flip induced relaxation and the precession of the spin accumulation imposed by noncollinear magnetizations.

The paper is organized as follows. Section II is devoted to the introduction of the BL equation for noncollinear transport and deriving the diffusion equations. In Sec. III we calculate all possible correlations of intrinsic fluctuations and Langevin sources. Section IV illustrates the formalism by calculating the shot noise for a double barrier FNF system. We show results for the charge current shot noise in Sec. V and finally give some conclusions in Sec. VI.

II. BOLTZMANN-LANGEVIN EQUATIONS FOR NONCOLLINEAR TRANSPORT

In this section we develop a semiclassical BL formalism for fluctuations of the noncollinear spin-polarized current in a diffusive normal metal connected through tunnel contacts to a number of ferromagnetic reservoirs. When the magnetization vectors of the reservoirs have a noncollinear orientation, the spin accumulation vector in the normal metal will also have a noncollinear direction with respect to the quantization axis (z). The semiclassical electronic distribution is then determined by a $2 \times 2$ matrix in the Pauli spin space of the form

$$f(k, \mathbf{r}, t) = \begin{pmatrix} f_1(k, \mathbf{r}, t) & f_1^\ast(k, \mathbf{r}, t) \\ f_1^\ast(k, \mathbf{r}, t) & f_1(k, \mathbf{r}, t) \end{pmatrix}. \quad (2.1)$$

The fluctuating distribution function matrix $\widehat{f}(k, \mathbf{r}, t)$

$$\widehat{f}(k, \mathbf{r}, t) = \widehat{f}(k, \mathbf{r}) + \delta \widehat{f}(k, \mathbf{r}, t),$$

which consists of the average distribution plus the time dependent fluctuations, depends on the momentum $k$, the coordinate $\mathbf{r}$ and time $t$. It is convenient to expand $\hat{f}$ into the charge and the spin vector distributions in terms of the $2 \times 2$ unit matrix and the Pauli matrices $\{1, \sigma\}$. The matrices for two distributions whose spin vector distributions are pointed in opposite directions take the form

$$\hat{f}(k, \mathbf{r}) = \hat{f}_c(k, \mathbf{r}, t) \hat{1} \pm \hat{\sigma} \cdot \hat{f}_s(k, \mathbf{r}, t).$$

(2.2)

Here $\hat{f}_c = (f_1 + f_1^\ast)/2$ is the charge or the spin independent part of the distribution matrix and $\hat{f}_s$ is the spin distribution vector. The $z$ component of the spin distribution vector $f_{sz} = (f_1 - f_1^\ast)/2$ is a spin polarization along the quantization axis and the other two components, $f_{sx}$ and $f_{sy}$, describe the spin-polarization oriented perpendicular to the quantization axis. Note that the two distributions $\hat{f}_s$ are transformed to each other by making the replacement $\hat{f}_s \rightarrow -\hat{f}_s$.

The BL equation for the nonlinear distribution matrix (2.1) is written as follows:

$$\frac{d}{dt} \hat{f}_s = \hat{\Pi}^{imp}(\hat{f}_s) + \hat{\Pi}^{sf}(\hat{f}_s, \hat{f}_c) + \hat{\xi}^{imp} + \hat{\xi}^{sf}, \quad (2.3)$$

where $\hat{\Pi}^{imp}$ is the collision integral for normal spin-independent impurity (spin-flip) scattering and $\hat{\Pi}^{imp(sf)}$ is the corresponding Langevin source of the current fluctuations. The matrix collision integrals are expressed as the following:

$$\hat{\Pi}^{imp}(\hat{f}_s) = \int d\mathbf{k}' W^{imp}(\mathbf{k}, \mathbf{k}') [\hat{f}_s^\ast(\mathbf{k}') - \hat{f}_s(\mathbf{k})],$$

(2.4)

$$\hat{\Pi}^{sf}(\hat{f}_s, \hat{f}_c) = \int d\mathbf{k}' W^{sf}(\mathbf{k}, \mathbf{k}') [\hat{f}_s^\ast(\mathbf{k}') - \hat{f}_s(\mathbf{k})],$$

(2.5)

where $W^{imp}(\mathbf{k}, \mathbf{k}')$ is the rate of the impurity scattering in which an electron scatters from the state with momentum $\mathbf{k}$ into $\mathbf{k}'$ without changing its spin state. The spin-flip scattering rate $W^{sf}(\mathbf{k}, \mathbf{k}')$ describes the transition from the state with momentum $\mathbf{k}$ and spin state $|\uparrow, \pm\rangle$ to $\mathbf{k}'$ and the flipped spin state $|\downarrow, \mp\rangle$. The up and down spin eigenstates in the quantization axis parallel to the local spin-polarization vector $\mathbf{\sigma} = [\hat{f}_s^\ast] \mathbf{\sigma}$. In writing the expressions (2.4) and (2.5) we assumed that $W^{imp(sf)}(\mathbf{k}, \mathbf{k'}) = W^{imp(sf)}(\mathbf{k'}, \mathbf{k})$, $W^{imp(sf)}(\mathbf{k}, \mathbf{k'}) = W^{imp(sf)}(\mathbf{k'}, \mathbf{k})$. 
which follows from the detail balance principle. We also ignored dependence of the scattering rates on the spin state of electron.

The Langevin sources are expressed in terms of the fluctuating part of the distribution matrix as follows:

\[ \hat{\xi}^{\text{imp}} = \int d\mathbf{k}' W^{\text{imp}}(\mathbf{k}, \mathbf{k}')[\delta \hat{f}^r(\mathbf{k}') - \delta \hat{f}^s(\mathbf{k})], \]

\[ \hat{\xi}^{\text{sf}} = \int d\mathbf{k}' W^{\text{sf}}(\mathbf{k}, \mathbf{k}')[\delta \hat{f}^r(\mathbf{k}') - \delta \hat{f}^s(\mathbf{k})]. \]

(2.6)

(2.7)

We apply the standard diffusive approximation for a diffusive normal metal. Assuming that all quantities are sharply peaked around the Fermi level, the momentum vector \( \mathbf{k} \) is expressed in terms of the energy \( \varepsilon \) and the direction of the Fermi momentum \( \mathbf{n} \). Then the following relations are held for the elastic scattering of electrons by the normal impurities and the spin-flip disorders:

\[ W^{\text{imp}}(\mathbf{k}, k') = \frac{2}{N_0} \delta(\varepsilon - \varepsilon') W^{\text{imp}}(\mathbf{n}, \mathbf{n}'), \]

\[ W^{\text{sf}}(\mathbf{k}, k') = \frac{2}{N_0} \delta(\varepsilon - \varepsilon') W^{\text{sf}}(\mathbf{n}, \mathbf{n}'), \]

where \( N_0 \) is the density of states in the Fermi level. In the diffusive regime the electronic distribution is weakly anisotropic in the momentum space and can be expanded up to the linear term in \( \mathbf{n} \):

\[ \hat{f}(\mathbf{n}, \varepsilon, \mathbf{r}, t) = \hat{f}_0(\varepsilon, \mathbf{r}, t) + \mathbf{n} \cdot \hat{\mathbf{f}}(\varepsilon, \mathbf{r}, t). \]

(2.9)

The anisotropic part of the distribution matrix is given by the vector \( \hat{\mathbf{f}} \), whose components are 2 \( \times \) 2 matrices in the spin space. The current density matrix is expressed in terms of it as follows:

\[ \hat{\mathbf{j}}(\mathbf{r}, t) = \frac{\mathbf{e}}{\tau_{\text{sf}}} \int d\varepsilon d\mathbf{f} \hat{\mathbf{f}}, \]

(2.10)

where \( \nu_F \) is the Fermi velocity. The isotropic part \( \hat{f}_0 \) determines the electrochemical potential matrix

\[ \phi(\mathbf{r}, t) = \bar{\phi}(\mathbf{r}) + \delta \phi(\mathbf{r}, t) = (1/e) \int d\varepsilon \hat{f}_0. \]

(2.11)

Replacing expansion (2.9) in Eq. (2.3) we obtain diffusion equations for the current density and the electrochemical potential matrices, which read

\[ \nabla^2 \bar{\phi}(\mathbf{r}) = \frac{1}{\ell_{\text{sf}}^2} \left( \bar{\phi} - \frac{\text{Tr}(\bar{\phi})}{2} I \right), \]

\[ \mathbf{j} = - e^2 \frac{N_0}{2 \tau_{\text{sf}}} \left( \bar{\phi} - \frac{\text{Tr}(\bar{\phi})}{2} I \right) + \mathbf{\sigma} \cdot \mathbf{\zeta}, \]

\[ \hat{\mathbf{j}} = - \mathbf{\sigma} \nabla \phi + \hat{\mathbf{\eta}}. \]

(2.12)

(2.13)

(2.14)

In these equations \( \sigma = e^2 (N_0/2) \) is the conductivity of the normal metal and \( \ell_{\text{sf}} = \sqrt{D \tau_{\text{sf}}} \) is the spin-flip scattering length, where the diffusion constant is given by \( D = \nu_F^2 \tau_{\text{sf}} / 3 \). The impurity and the spin-flip relaxation times are defined by the following relations:

\[ \frac{n}{\tau_{\text{imp}}} = \int d\mathbf{n}' W^{\text{imp}}(\mathbf{n}, \mathbf{n}')(\mathbf{n} - \mathbf{n}'), \]

\[ \frac{n}{\tau_{\text{sf}}} = 2 \int d\mathbf{n}' W^{\text{sf}}(\mathbf{n}, \mathbf{n}')(\mathbf{n} - \mathbf{n}'), \]

\[ \frac{1}{\tau} = \frac{1}{\tau_{\text{imp}}} + \frac{1}{\tau_{\text{sf}}}. \]

(2.15)

(2.16)

(2.17)

The Langevin term for the current fluctuations matrix in Eq. (2.14) has the form

\[ \hat{\mathbf{\eta}} = e \nu_F (N_0/2) \tau \int d\varepsilon d\mathbf{n} \left( \hat{\xi}^{\text{imp}} + \hat{\xi}^{\text{sf}} \right) d\mathbf{e}. \]

(2.18)

Also,

\[ \mathbf{\dot{\sigma}} \cdot \mathbf{\zeta}(\mathbf{r}, t) = (e N_0 / 2) \int d\varepsilon d\mathbf{n} \left( \hat{\xi}^{\text{sf}} - \frac{\text{Tr}(\hat{\xi}^{\text{sf}})}{2} I \right), \]

(2.19)

is the Langevin term for divergence of the current fluctuations matrix due to the nonconserved nature of the spin-flip scattering. These two Langevin terms of fluctuations have zero average values.

Note that we disregarded the explicit dependence of \( \hat{f} \) on time, given by the term \( \partial \hat{f} / \partial t \) in the left-hand side of the BL equation, since we are interested in the zero frequency noise power. To obtain these equations we focused on the realistic limit when \( \tau_{\text{imp}} \ll \tau_{\text{sf}} \) and ignored the effect of the spin-flip scattering on the conductivity of the diffusive normal metal.

III. CORRELATIONS OF THE CURRENT FLUCTUATIONS

A. Correlations of the Langevin terms of current fluctuations

To calculate correlations between the components of the Langevin terms of the fluctuations \( \hat{\mathbf{\eta}} \) and \( \mathbf{\zeta} \) we transform to a new spin basis, where the spin quantization axis is parallel to the local mean spin distribution vector \( \mathbf{s} = \frac{1}{N_0} \int d\varepsilon \mathbf{s} \). Denoting the projection matrices along \( \mathbf{s} \) and two perpendicular unit vectors \( \mathbf{s}_i,i (i = 1, 2) \) by \( \mathbf{v}_i = \mathbf{\sigma} \cdot \mathbf{s} \) and \( \mathbf{v}_{i-} \), respectively, the distribution function matrix can be expanded as follows:

\[ \hat{\mathbf{j}}^n = \frac{1}{2} (f_{+\mathbf{s}} + f_{-\mathbf{s}}) I + \frac{1}{2} (f_{+\mathbf{s}} - f_{-\mathbf{s}}) \mathbf{v}_+ \pm 2 \sum_i (\delta f_{1,+i} - \delta f_{1,-i}) \mathbf{v}_{i-}, \]

(3.1)

where we have denoted the transverse components of the spin polarization vector by \( \delta \), which refers to the fluctuations, since they have zero mean values.

We substitute the fluctuating parts of the matrices (3.1) into the Langevin source terms (2.6) and (2.7) and obtain the result.
In these equations we have shown the current in individual from that we obtain all possible correlations between the matrix elements of the Langevin sources and assume that all scattering events are independent elastic scatterings as

\[ J_{k,k'} = J^{\text{imp}}(k,k') - J^{\text{imp}}(k,k') \cdot \tilde{s}_{\text{imp},d} \]

In these equations we have shown the current in individual impurity (spin-flip) scattering as

\[ J^{\text{imp}}(k,k') = W^{\text{imp}}(k.k') \times (k,k')f_{\alpha,\alpha}(k)[1-f_{\alpha,\alpha}(k')] \]

for a component of spin in the direction of \( \hat{s} \).

Now we apply the central assumption of the BL approach and assume that all scattering events are independent elementary processes and thus the correlations of the associated currents fluctuations obey the Poissonian relation:

\[ \langle \delta J_{k_{1},k_{2},r_{1},t}^{\text{imp}}(k_{3},k_{2},r_{1},t') \rangle = \delta_{ij} \delta_{k_{1},k_{2}} \delta_{k_{3},k_{1}} \delta(t-t') \]

\[ \times \delta(\mathbf{r}-\mathbf{r}') \right) \tilde{\rho}_{\text{imp}}(k_{1},k_{2},r_{1},t) \]

Using these relations we can calculate the correlations between the matrix elements of the Langevin sources (\( \xi^{\text{imp}}(i) \)).

From that we obtain all possible correlations between components of the vector matrices \( \hat{y} \) and \( \xi \). The results read

\[ \langle \eta^{\alpha\beta}(r,t) \eta^{\gamma\delta}_{m}(r',t') \rangle = \frac{1}{2} \delta_{ij} \delta(\mathbf{r}-\mathbf{r}') \delta(t-t') \sigma \]

\[ \times \left\{ \delta_{ij} \delta(\mathbf{r}-\mathbf{r}') \delta(t-t') \right\} \]

\[ \langle \xi^{\alpha}(r,t) \xi^{\gamma}(r',t') \rangle \]

\[ = \delta(\mathbf{r}-\mathbf{r}') \delta(t-t') \sigma \]

\[ \times \frac{1}{D_{\text{sd}}} \tilde{\rho}_{\text{sd}} \sum_{\nu} \Pi_{\nu,\nu}(r) \]

\[ \langle \eta^{\alpha\beta}(r,t) \tilde{\eta}^{\gamma}(r',t') \rangle = 0, \]

where

\[ \tilde{f}_{\alpha,\alpha}(r) = \int d\mathbf{r}' \tilde{f}_{\alpha,\alpha}(r)[1-\tilde{f}_{\alpha,\alpha}(r')] \]

Here \( \tilde{f}_{\alpha,\alpha}(r) \) is the isotropic part of the up and down spin components of the mean distribution function with respect to the quantization axis parallel to \( \hat{s} \). So, if we know the mean distribution function matrix we can easily calculate these correlations. It is a remarkable result that, just the longitudinal fluctuations of the distribution function have a nonvanishing contribution to correlations and hence in shot noise. But, the transverse fluctuations of the distribution function actually make no contribution in the shot noise.

### B. Boundary conditions and correlations of the intrinsic fluctuations

The diffusion equations (2.12)–(2.14) and Eqs. (3.5)–(3.7) are the BL equations for the noncollinear spin current. These equations are extension of the BL equations obtained in Ref. 21 for the collinear spin-polarized current. They are a complete set of equations, which have to be implemented by the appropriate boundary conditions at the contacts of the normal metal to the ferromagnetic terminals. For a FN junction with a noncollinear spin current the expression for the current matrix reads:

\[ e \bar{F} = G^{\uparrow\downarrow}[\tilde{f}_{\uparrow} - \tilde{f}_{\downarrow}] \hat{u}^\dagger + G^{\uparrow\downarrow}[\tilde{f}_{\uparrow} - \tilde{f}_{\downarrow}] \hat{u}^\dagger \]

\[ -G^{\uparrow\downarrow} \hat{u}^\dagger \tilde{f}_{\uparrow} \bar{N} \hat{u}^\dagger \]

The mixing conductance \( G^{\uparrow\downarrow} = \frac{\pi}{\Sigma_{nm}[r_{nm}^\dagger r_{nm}^\dagger]} \) describes the transfer of the spin current perpendicular to the magnetization of F. In this equation \( \hat{u}^{\dagger}(\hat{u}^\dagger) = \frac{1}{2}[1+(\hat{t} \cdot \hat{n})] \) are projection matrices in the spin space in which \( \hat{n} \) is the unit vector in the direction of the magnetization vector of F.

To write an expression for the fluctuations of the current matrix (3.9) we follow the Beenakker and Büttiker assumption and consider that the time-dependent fluctuations have two contributions. The first contribution \( \delta(\hat{I}) \) is the intrinsic fluctuations of the current matrix, which is caused by the randomness of the electron scattering from the junction. This is the only relevant term for the shot noise of a single junction connecting two reservoirs which are held at equilibrium. The second contribution can exist when the distribution functions in N metal and/or the adjacent reservoirs are fluctuating. For a contact between a N metal with fluctuating part of the distribution matrix \( \delta f_{N} \) and a F reservoir held at equilibrium, the expression for the current fluctuations matrix has the form
The correlations of the intrinsic fluctuations for an arbitrary type of a FN contact with a noncollinear spin current was calculated by Tserkovnyak and Brataas\textsuperscript{24} using a generalized LB approach. Here we use their result and write the correlations between the different elements of the intrinsic current fluctuations matrix $C_{\alpha\beta\alpha'\beta'} = \int dt (\delta I^\alpha (t) \delta I^{\alpha'} (0))$, as follows:

$$
\begin{align*}
&\delta I^\alpha = -G^{1\dagger} \delta \mathcal{F}^\dagger_{\text{N}} (\mathbf{r}) \delta I^\alpha (\mathbf{r}) - G^{1\dagger} \delta \mathcal{F}^{\dagger}_{\text{N}} (\mathbf{r}) \delta I^\alpha (\mathbf{r}) \\
&\quad - G^{1\dagger} \delta \mathcal{F}^\dagger_{\text{N}} (\mathbf{r}) \delta I^{\alpha'} (\mathbf{r}) - (G^{1\dagger})^* \delta \mathcal{F}^{\dagger}_{\text{N}} (\mathbf{r}) \delta I^{\alpha'} (\mathbf{r}).
\end{align*}
$$

\text{(3.10)}

The general solution of this equation can be written as the energy integral of the following relation:

$$
\tilde{f}_N^\dagger (x) = \left[ f_1^\dagger + (f_2^\dagger - f_1^\dagger) \left( a + \frac{b}{L} \right) \right] \hat{I} + \mathbf{\tilde{a}} \cdot [\tilde{c} \sinh(\lambda x/L) + \tilde{d} \cosh(\lambda x/L)] (f_2^\dagger - f_1^\dagger),
$$

\text{(4.1)}

where $\lambda = L/\ell_{sd}$ is a dimensionless parameter which measures the intensity of the spin-flip scattering. The distribution function matrices at the terminals $F_1$ and $F_2$ are determined by the Fermi distribution functions via $f_1^\dagger = f_{\text{FP}} (a) \hat{I}$ and $f_2^\dagger = f_{\text{FP}} (a - eV) \hat{I}$. The scalars $a, b$ and components of the vectors $\tilde{c} = (c_1, c_2, c_3)$ and $\tilde{d} = (d_1, d_2, d_3)$ are coefficients which have to be determined by the boundary conditions. The boundary conditions are the spin-charge current conservation rules which in the absence of the spin-flip process at the tunnel contact $i$ ($i=1,2$), read

$$
\tilde{I}_i^c + \tilde{I}_i^N = 0,
$$

\text{(4.2)}

We consider a two terminal diffusive spinvalve as shown in Fig. 1. Two ferromagnetic reservoirs $F_1$ and $F_2$ with a noncollinear orientation $\theta$ of the magnetization vectors are connected through tunnel contacts to a diffusive normal metal wire (N) of length $L$. The reservoirs $F_{1,2}$ are held at 0 and $V$ potentials, respectively. We consider a symmetric structure for which both of the tunnel contacts have the same conductance and shot noise matrices given by Eqs. (3.12) and (3.13).

The mean distribution function matrix inside the N metal is obtained by solving the diffusion equation (2.12). The general solution of this equation can be written as the energy integral of the following relation:
where the normal conductance of the N wire is given by
\[ G_N = \sigma A / L, \] A is the cross section area of the wire.

To obtain correlations of the Langevin sources in Eqs. (3.5)–(3.7) we need the distribution function matrix in the quantization axis parallel to the mean spin distribution vector. This is achieved by diagonalizing the distribution function matrix to get \( \tilde{f}_{\ell,s}(x) \). The result is
\[
\tilde{f}_{\ell,s}(x) = f_1 + (f_2 - f_1) \left[ \left( a + b \frac{x}{L} \right) \pm \bar{c} \sinh(\lambda x/L) \right.
+ \bar{d} \cosh(\lambda x/L) \right],
\]

where \( f_{1,2} \) are the corresponding Fermi distributions in the terminals. Now we calculate the fluctuations of the current matrix \( \hat{\Delta}^N \) at the contacts \( x=0, L \) inside the N metal. Using the equation for \( \hat{\Delta} \) (2.14) and Eq. (2.13) we obtain
\[
\hat{\Delta}^N(0,L) = -\sigma \oint ds \left[ \nabla \phi_{\ell \ell}(x) \frac{1}{2} \left( \hat{\rho} \phi_{\ell \ell}(x) \right) + \nabla \phi_{\ell \ell}(x) \right]
\times \left( \hat{\rho} - \frac{1}{2} \right) + \hat{\delta}^N(0,L),
\]

where integration is over the surface of the N metal and
\[
\hat{\delta}^N(0,L) = \int dx \frac{1}{2} \left( \hat{\rho} \cdot \nabla \phi_{\ell \ell}(x) \right) \frac{1}{2}
+ \left( \nabla \phi_{\ell \ell}(x) \right).
\]

Here the potential functions are defined
\[
\phi_{\ell \ell}(x) = 1 - \frac{x}{L},
\]
\[
\phi_{\ell \ell}(x) = \frac{x}{L},
\]
\[
\phi_{\ell \ell}(x) = \frac{\sinh(\lambda x/L)}{\sinh(\lambda)}.
\]

Note that as a result of the spin-flip scattering the spin current and, hence, its fluctuations are not conserved through the wire.

Now we can apply the current conservation law for the fluctuations of the currents matrix at two contacts:
\[
\hat{\Delta}^N_l + \hat{\Delta}^N_r = 0.
\]

Replacing the expressions of the current fluctuations from Eqs. (4.5)–(4.10) and Eq. (3.10) in Eq. (4.11) we obtain a system of eight linear equations whose solutions give the fluctuations of the chemical potential matrix \( \hat{\delta}^N_l(0,L) \) at the points \( x=0, L \) in terms of the intrinsic current fluctuations matrix \( \hat{\delta}^N_{1,2} \) and the Langevin terms of the current fluctuations matrix \( \hat{\delta}^N_l(0,L) \). Replacing this result into Eq. (3.10) the current fluctuations matrix is expressed in terms of the Langevin terms and the intrinsic current fluctuations matrices as follows:
\[
\hat{\delta}^N = \sum_{\alpha'\beta'} \left[ A_{\alpha'\beta'} \hat{\delta}^N_{\alpha'\beta'} + B_{\alpha'\beta'} \hat{\delta}^N_{\alpha'\beta'} + C_{\alpha'\beta'} \hat{\delta}^N_{\alpha'\beta'} \right] (0)
+ D_{\alpha'\beta'} \hat{\delta}^N_{\alpha'\beta'} (L),
\]

where \( A_{\alpha'\beta'}, B_{\alpha'\beta'}, C_{\alpha'\beta'}, \) and \( D_{\alpha'\beta'} \) are the coefficients which depend on \( G/G_N, P, \lambda, \) and \( \theta \). From this result we can calculate the correlations of the charge current \( S = 2 \int d\tau \langle \Delta l \Delta L \rangle \) and the corresponding Fano factor \( F = S / 2 e \bar{I} \). The resulting expressions are too lengthy to be written down here. In the next section we analyze the magnetostatic noise, the dependence of \( F \) on the relative angle \( \theta \), for different values of the involved parameters.

**V. RESULTS AND DISCUSSIONS**

Let us start our analysis of the shot noise with the limiting case of negligible spin-flip scattering, \( \lambda \to 0 \) and highly resistive tunnel contacts, \( G/G_N \to 0 \). In this limit we retrieve the results of Tserkovnyak and Brataas,\(^{24}\) which is a monotonic variation of the Fano factor as a function of the relative angle of magnetization vectors \( \theta \):
\[
F = 1 - \frac{1}{2} \left[ 1 + P^2 \sin^2(\theta/2) \right].
\]

For a finite \( G/G_N \) the angular dependence of the Fano factor deviates from the above simple monotonic behavior. This is illustrated in Fig. 2, where we have plotted \( F(\theta) \) for different values of \( G/G_N \) in the limit \( \lambda \to 0 \) and the polarization \( P \) \( =0.99 \). When \( G/G_N \) increases, \( F \) decreases below the value given by Eq. (5.1) and finds a nonmonotonic dependence on \( \theta \).
For antiparallel magnetization vectors θ = π the Fano factor \(F\) reaches to the Poissonian value one due to the large collinear spin accumulation in the N metal. For parallel magnetizations θ = 0, this collinear spin accumulation is suppressed and, consequently, the shot noise is reduced to the value close to the normal state case. When the magnetization vectors have noncollinear orientation the spin accumulation vector undergoes a spatial precession inside the N metal. The total precession angle, i.e., the angle between spin accumulation directions at the two contact points, depends on θ and \(G/G_N\). We note that \(G/G_N\) determines the miss orientation of the spin accumulation vector at a contact point from the magnetization vector of the connected F reservoir. The noncollinear configuration of the spin polarization inside the N metal produces additional correlations between the electronic spin states. These spin-dependent correlations, which increase with increasing the total precession angle, can impose further restrictions on the scattering of electrons from impurities, and consequently decrease the shot noise. On the other hand increasing θ from 0 increases the amplitude of the spin accumulation in N, which tends to increase the shot noise. The nonmonotonic dependence of \(F\) on θ is the result of the competition of these two effects. As a result the Fano factor develops a minimum at a finite \(θ\) which depends on \(G/G_N\) and \(P\).

In the limit of \(G/G_N \ll 1\), \(F\) returns back to the constant normal state value 1/3 as expected. This holds for not perfectly polarized contacts \(P \neq 1\). We found that in the special case of \(P = 1\), \(F\) retains its nonmonotonic angular dependence in the limit \(G/G_N \gg 1\). In this case \(F\) has a minimum well below 1/3 at a finite \(θ\), a sharp peak at \(θ = π\) where it reaches one and a smooth peak at \(θ = 0\) with the value 1/3. There is a corresponding behavior in the conductance of the system in this limit.\(^{12}\)

To show the effect of the contacts polarization \(P\), in Fig. 3 we have plotted \(F(P)\) for different \(θ\), in the limit of \(λ \rightarrow 0\) and when \(G/G_N = 5\). For \(P = 0\) the Fano factor takes the normal state value, irrespective of the value of \(θ\).

When we increase \(P\), the angular dependence of \(F\) appears. The Fano factor increases (decreases) monotonically with \(P\) for \(θ = π (\theta = 0)\) and reaches a maximum (minimum) at \(P = 1\). However, intermediate angles \(F(P)\) can have a nonmonotonic variation with a maximum at \(P < 1\). This is in contrast to the limit of the normal metal node geometry \((G/G_N \rightarrow 0)\), where a monotonic dependence on \(P\) was predicted [Eq. (5.1)].

Next we study the effect of the spin-flip scattering in the N metal. In Fig. 4 we show the dependence of the Fano factor on \(λ\) for different angles, when \(G = 1\) and \(P = 1\). As it can be seen from Fig. 4, a strong spin-flip scattering destroys any spin polarization and thus suppresses the dependence of the Fano factor on \(θ\). In this limit \((λ \gg 1)\) \(F\) reduces to the normal state value [Eq. (5.2)], independent of \(θ\). At a finite \(λ\) the shot noise of different \(θ\)s are separated. When \(λ\) increases from zero, \(F(λ)\) passes through a minimum or maximum, depending on \(θ\), before reaching the normal state value at large \(λ\)s. While the minimum and maximum values of \(F\) are located at \(λ = 0\) for \(θ = 0\) and \(θ = π\), they can happen at finite \(λ\)s for intermediate angles. This emphasizes the pronounced effect of the spin-flip scattering on the shot noise.

VI. CONCLUSIONS

In this paper we have presented a semiclassical theory of the spin-polarized current fluctuations in a diffusive normal metal which is connected by tunnel contacts to ferromagnetic terminals with noncollinear magnetizations. Based on the Boltzmann-Langevin approach, we have developed diffusion equations which allow for the calculation of the charge-spin distribution and current density matrices and correlations of the corresponding fluctuations in noncollinear multiterminal systems. Our theory takes into account the spatial precession of the noncollinear spin accumulation as well as the spin-flip induced relaxation in the normal metal. Applying the developed theory to a two terminal FNF structure we have found...
that the Fano factor has a nonmonotonic dependence on the magnetization angle, provided that the spin-flip intensity is small and the tunnel contacts have an appreciable polarization. For antiparallel orientation the Fano factor is found to increase well above the unpolarized value due to a large spin accumulation in the N metal. In contrast, for the parallel orientation the shot noise is almost identical with its normal state (unpolarized) value because of the spin accumulation suppression. At the intermediate angles we found that the interplay between the spin accumulation precession and suppression with the magnetization angle causes the Fano factor to develop to a minimum. The minimum shot noise can be substantially below the normal value depending on the relative conductances of the N metal and tunnel contacts $G/G_N$ and the polarization $P$. We have shown further that the spin-flip scattering diminishes the amplitude of the nonmonotonic behavior as well as the polarization effects.

In spite of a few early25,26 and recent27 experimental studies devoted to the spin-polarized shot noise in magnetic tunnel junctions between ferromagnets, to our knowledge, there have not been reports on shot noise measurements in metallic spin-valve systems. We expect that such experiments will be performed in the near future, which will reveal the effects predicted in the present paper.

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