Surface polaritons of one-dimensional photonic crystals containing graphene monolayers

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Abstract
We investigated theoretically the existence of surface polaritons (SPs) at the interface of a one-dimensional photonic crystal containing graphene monolayers. It is shown that the structure has a new type of the photonic band gap in the THz region which is strictly omnidirectional for the TM-polarization and can support the SPs for both TM-polarization and TE-polarization. The results show that the characteristics of the SPs depends on the optical properties of the graphene sheets which can be controlled by a gate voltage. We plotted the electromagnetic field profiles of the SPs at the frequency range of the graphene induced band gap and a conventional Bragg gap of the structure. It is found that the SPs at the graphene induced band gap are more localized than the SPs at the Bragg gaps.

1. Introduction

Surface polaritons (SPs) are a kind of electromagnetic normal modes, which propagate along the interface and have an evanescent behavior both sides of the interface with their electric and magnetic fields localized near the interface. The physical origin of the SPs are similar to that of the surface plasmon polaritons (SPPs) which exist at the interfaces of metal films. So, these modes can be considered to be an optical analog of the SPPs [1]. From this point of view, the SPs are very sensitive and convenient tools for studying the physical properties of the surfaces. Therefore, the investigations of SPs are...
important from the scientific point of view and the practical applications [2]. During the past decade, SPs supported by semi-infinite one dimensional photonic crystals (1D PCs) have gained much attention due to their potential capability for many applications such as sensors, fluorescence emission enhancement, enhancement of the Goos-Hanchen shift [3–7]. This kind of SPs can be excited in a broad range of frequency by properly choosing the optical and geometrical parameters of the PC and modifying its photonic band gaps (PBGs). This capability can be considered as an advantage of the SPs comparing with the SPPs.

Until now, the optical properties of the PCs containing various kinds of materials including dielectrics, metals, semiconductors and metamaterials have been investigated [8–11]. Besides, the excitation of the SPs at the interfaces of the 1D PCs containing metallic layers, single-negative or double-negative metamaterials have been studied extensively [11–16]. Meanwhile, it is important to control the dispersion properties of the SPs by tuning the PBG of the PCs. This can be possible by controlling the optical properties of the constituent materials. So, it is highly appropriate to use the materials with externally controllable optical properties. In this regard, graphene monolayers can be a suitable option for using in the 1D PC structures.

Graphene, a single two-dimensional plane of carbon atoms forming a honey-comb lattice, is a gapless semiconductor with controllable electronic and optical properties [17]. High mobility of carriers, flexibility, robustness and environmental stability are the general features of the graphene [18,19]. At the THz and far-IR frequencies, the dissipative losses of the graphene is less than the usual metals and its electronic and also optical response is described by the surface conductivity which is related to its chemical potential and can be controlled and tuned by the external electric fields [20,21]. Due to these unique characteristics, people have been motivated to study the multilayer structures containing graphene sheets. For example, THz hyperbolic metamaterials and hyperlenses made of graphene multilayers [22–24] have been investigated, recently. Also, the excitation of the SPs in some kinds of graphene-based layered structures have been studied and experimentally realized by different research groups. As examples, Liu et al. showed that the absorption of the graphene on the top of a 1D PC is enhanced greatly due to photon localization [25]. Sreekanth et al., experimentally demonstrated the excitation of the surface waves in a graphene-based Bragg grating using prism coupling technique [26]. In another research, Sreekanth et al. investigated the propagation length of the plasmon waves in a graphene monolayer sandwiched by two identical anisotropic dielectrics [27], Wang et al. theoretically studied the coupling between SPPs in monolayer graphene sheet arrays [28].

In Ref. [29], the authors investigated the PBGs of a 1D PC in which the graphene sheets were embedded between adjacent dielectric layers. They revealed that the structure has a new type of the band gap (the so-called graphene induced photonic band gap (GIPBG) in the THz region which is omnidirectional only for the TM-polarization; and its characteristic properties is different from the conventional Bragg gaps. In the present paper, we are interested to study the dispersion properties of the SPs at the interface of a semi-infinite uniform dielectric medium and a 1D PC containing graphene monolayers in the GIPBG. In our numerical analysis, we use the well known transfer matrix method [1] to calculate the PBGs of the 1D PC and the dispersion curves of the SPs. We show that the dispersion properties of the SPs can be controlled by tuning the chemical potential of the graphene sheets. Moreover, it is shown that the SPs at the GIPBG are more localized than the SPs at the Bragg band gaps.

2. Theoretical model

In this section we obtain the dispersion relation of the SPs propagating along the $x$ axis situated at the interface between a dielectric uniform medium with the permittivity of $\varepsilon_v$ and a semi-infinite 1D PC. The 1D PC consists of alternate layers $A$ and $B$ with the graphene monolayers between the adjacent layers. Here, $A$ and $B$ represent two isotropic dielectric materials with the permittivity of $\varepsilon_A$ and $\varepsilon_B$, and thicknesses of $d_A$ and $d_B$, respectively (Fig. 1). We assume that the PC is capped by a layer of the same material $A$ but different width, $d_c$. We choose the layers to be parallel to the $x-y$ plane with the $z$ axis normal to the interfaces of the layers. The general expression for the surface conductivity of the graphene monolayers can be written as [30]
\[
\sigma_g(\omega) = \frac{e^2 \omega}{\pi \hbar} \left[ \int_{-\infty}^{+\infty} \frac{d\xi}{\omega^2} \frac{|\xi| df(\xi)}{d\xi} - \int_{0}^{+\infty} d\xi \frac{f(-\xi) - f(\xi)}{(\hbar \omega + i \Gamma)^2 - 4\xi^2} \right] 
\]

where \( f(\xi) = \left[ \exp\left((\xi - \mu_c)/k_B T\right) \right]^{-1} \) is the Fermi function and \( \Gamma \) is the scattering rate. At room temperature, \( T = 300 \text{ K} \), the value of \( \Gamma \) is in the range of 0.1 meV [23], and for the frequencies in the range of THz (\( \hbar \omega \gg \Gamma \)), \( \sigma_g \) can be obtained as
\[
\sigma_g(\omega) = \sigma_g^{\text{intra}}(\omega) + \sigma_g^{\text{inter}}(\omega) [30],
\]
where
\[
\sigma_g^{\text{intra}}(\omega) = \frac{e^2}{4\hbar} \frac{i}{2\pi} \left\{ \frac{16k_B T}{\hbar \omega} \ln \left( \frac{2 \cosh \left( \frac{\mu_c}{2k_B T} \right) \right) \right\},
\]
\[
\sigma_g^{\text{inter}}(\omega) = \frac{e^2}{4\hbar} \left\{ \frac{1}{2} + \frac{\pi}{2} \arctan \left( \frac{\hbar \omega - 2\mu_c}{2k_B T} \right) - \frac{i}{2\pi} \ln \left( \frac{(\hbar \omega + 2\mu_c)^2}{(\hbar \omega - 2\mu_c)^2 + (2k_B T)^2} \right) \right\}.
\]

Here, \( e \) is the charge of the electron, \( k_B \) is the Boltzmann constant, \( T \) is the temperature and \( \mu_c \) is the chemical potential which can be controlled by applying a gate voltage. As it is clear from these expressions, the interband contribution is important above the absorption threshold, \( \hbar \omega \approx 2\mu_c \), while it is negligible comparing with the intraband contribution at relatively low frequencies, \( \hbar \omega < \mu_c \).

We assume that the tangential components of the electromagnetic fields of the TM-polarized SPs are given as
\[
\tilde{H}_{1y} = \hat{e}_y b_x e^{i k_0 z} e^{i(kx - \omega t)},
\]
\[
\tilde{E}_{1x} = \hat{e}_x \left( -\frac{i k q_0}{\omega \epsilon_0 \epsilon_0} \right) b_x e^{i k_0 z} e^{i(kx - \omega t)}
\]
in the region \( z < -d_c \) and
\[
\tilde{H}_{2y} = \hat{e}_y H_y(z) e^{i(kx - \omega t)},
\]
\[
\tilde{E}_{2x} = \hat{e}_x \left( -\frac{i}{\omega \epsilon_0 \epsilon(z)} \right) \frac{\partial H_y(z)}{\partial z} e^{i(kx - \omega t)}
\]
in the region \( z > -d_c \). Here \( k = (\omega/c) \), \( c \) is the speed of light and \( \beta \) is the normalized wavenumber along the interface. \( q_0 = \sqrt{\beta^2 - \epsilon_q} \) is the dimensionless transverse wavenumber which characterize the inverse decay length of the SPs in the uniform medium. We look for the SP modes correspond to the localized solutions with the field decaying from the interface in both directions and propagating along the interface. In the left-side uniform medium (see Fig. 1), the fields are decaying provided \( \beta^2 > \epsilon_q \), while in the right side periodic structure, the waves are the Bloch modes
\[
H_y(z) = \psi(z) e^{i k_0 z}
\]
where $K_b$ is the Bloch wavenumber, and $\psi(z)$ is the Bloch function, which is periodic with the period of the photonic structure. In the periodic structure the waves will be decaying provided $K_b$ is complex; and this condition defines the spectral gaps of an infinite photonic crystal which can be obtained by using the transfer matrix method \cite{1}. To find the dispersion properties of the SPs, we use Eq. (3) for the uniform medium and Eqs. (4) and (5) for the periodic structure and satisfy the conditions of continuity for the tangential components of the electric and magnetic fields at the interface between the uniform medium and the 1D PC structure. Finally, we arrive at the following dispersion relation for the TM-polarized SPs:

$$\left( \frac{q_A + ik\sigma_A q_A}{k q_A} \right) = i \frac{\omega}{\varepsilon_0} \left( \lambda - A + Be^{-2ikq_b d_c} \right)$$

where,

$$A = \frac{1}{4} e^{ikq_b d_c} \left\{ [1 + t_1 - t_2](1 + t_3 - t_4)e^{ikq_b d_a} + [(1 - t_1 + t_2)(1 - t_3 - t_4)e^{-ikq_b d_a}] \right\},$$

$$B = \frac{1}{4} e^{ikq_b d_c} \left\{ [(1 + t_1 - t_2)(1 - t_3 + t_4)e^{ikq_b d_a} + [(1 - t_1 + t_2)(1 - t_3 + t_4)e^{-ikq_b d_a}] \right\}$$

are the elements of the transfer matrix of the 1D PC and $\lambda = \text{Re}[A] \pm \sqrt{(\text{Re}[A])^2 - 1}$ is its eigenvalue. Here,

$$q_i = \sqrt{e_i - \beta^2}, \quad (i = A, B)$$

and

$$t_1 = \frac{\varepsilon_A q_B}{\varepsilon_B q_A}, \quad t_2 = \frac{\sigma_A kq_B}{\omega \varepsilon_0 e_B},$$

$$t_3 = \frac{\varepsilon_B q_A}{\varepsilon_A q_B}, \quad t_4 = \frac{\sigma_A kq_A}{\omega \varepsilon_0 e_A}.$$ 

For the TE-polarized SPs, the tangential components of the electromagnetic fields are assumed as:

$$\vec{E}_{1y} = \varepsilon_{r\varphi} b_{\varphi} e^{ikq_0 z} e^{(ikx - \omega t)},$$

$$\vec{H}_{1x} = \hat{\varepsilon}_x \left( \frac{ikq_0}{\omega \mu_0} \right) b_{\varphi} e^{ikq_0 z} e^{(ikx - \omega t)}$$

in the region $z < -d_c$ and

$$\vec{E}_{2y} = \varepsilon_{r\varphi} E_y(z) e^{(ikx - \omega t)},$$

$$\vec{H}_{2x} = \hat{\varepsilon}_x \left( \frac{i}{\omega \mu_0} \right) \frac{\partial E_y(z)}{\partial z} e^{(ikx - \omega t)}$$

in the region $z > -d_c$. In a similar way, one can obtain the following dispersion relation for the TE-polarized SPs:

$$\left( \frac{q_0 - i\sigma_A \omega \mu_0}{k q_A} \right) = -i \frac{\lambda - A + Be^{-2ikq_b d_c}}{\lambda - A - Be^{-2ikq_b d_c}}.$$ 

Here, $A, B, q_A$, and $q_B$ are given by Eqs. (7) and (8) with

$$t_1 = \frac{q_A}{q_B}, \quad t_2 = \frac{\omega \mu_0 \sigma_A}{k q_B},$$

$$t_3 = \frac{q_B}{q_A}, \quad t_4 = \frac{\omega \mu_0 \sigma_A}{k q_A}.$$
3. Results and discussion

In our numerical calculations, we take the optical and geometrical parameters of the system as follows: \(d_A = 10 \, \mu m\), \(d_B = 10 \, \mu m\), \(\varepsilon_A = 5\), \(\varepsilon_B = 2.5\) and \(\varepsilon_V = 1\). Due to the low loss of the graphene sheets in the THz and far-IR frequencies, we limit our analysis to the frequency range of 0 – 8 THz. Besides, the dispersion of the usual dielectric materials is not noticeable in this region [31]. Here, we assume \(\mu_c = 0.2 \, eV\) and \(T = 300 \, K\) to obtain the surface conductivity of the graphene sheets from Eq. (2). Fig. 2 represents the transmission of a graphene monolayer in the plane of the incidence angle (\(\theta\)) and the frequency (\(f\)). It’s evident from the figure that the transmission of the monolayer is strongly depends on the incidence angle and the polarization of the incident waves in the given frequency range. However, at a very small frequency interval \(f < 0.25 \, THz\), the transmission is near zero for the all incidence angles and both of the polarizations. The frequency region of zero transmission can be expanded by increasing the number of the graphene monolayers. Finally, this effect results in the creation of an omnidirectional PBG in the multilayer structures containing the graphene sheets (GIPBG).

To show this clearly, the photonic band structure of the graphene based 1D PC is represented in the plane of \((f - \beta)\) for the TE-polarized and TM-polarized waves in Fig. 3. Here, the PBGs are represented by the unshaded areas while the shaded areas show the allowed bands. As it’s clear from the figure, the structure has two PBGs in the given frequency range. The first PBG (the gap located at low frequencies around 1 THz) is a GIPBG [29] which is strictly omnidirectional for the TM-polarization. Omnidirectional PBGs can also be found in the metamaterial photonic superlattices (the so-called zero-\(n\) gap). In this regard, Agudelo-Arango et al. have been shown that the omnidirectional PBG condition can be satisfied for both the TM and TE-polarizations by appropriately choosing the thickness ratio of the PC layers [15]. Nevertheless, our considerations in the PBG properties of the graphene based 1D PC reveal that the variation of the ratio \(d_A/d_B\) does not affect the shape of the GIPBGs significantly and omnidirectional GIPBG can be obtained only for the TM-polarization. The major advantage of the GIPBG is the tunability of its width and its central frequency due to the controllable electronic and optical characteristics of the graphene [29]. On the other hand, the second PBG around 5 THz is polarization sensitive and depends strongly on the incidence angle which is the characteristic of the conventional Bragg gaps.

Now, we want to investigate the dispersion properties of the TM-polarized and TE-polarized SP modes at the first and second PBGs of the 1D PC. To do this, we numerically solved Eqs. (6) and (12) to obtain the dispersion curves of the TM-polarized and TE-polarized SP modes. Fig. 4 shows the results on the plane of the frequency \(f\) versus the propagation constant \(\beta\) for \(d_c = 0.1d_A\) (the dotted lines), \(d_c = d_A\) (the dashed lines) and \(d_c = 3d_A\) (the solid lines). As Fig. 4 reveals, the structure can support both TM-polarized and TE-polarized SP modes at the GIPBG. We call these SP modes as the

![Fig. 2](image.png)

Fig. 2. The transmission of a graphene monolayer in the plane of the incidence angle and the frequency for the TE and TM polarizations. Here, we assumed \(\mu_c = 0.2 \, eV\), \(T = 300 \, K\).
graphene induced surface polariton modes (GISP modes). The dispersion properties of the GISP modes depend on the value of $d_c$. For small $d_c$, we have only the TM-polarized GISP modes which are restricted to a limited range of $\beta$ (see the dotted lines). However, for large $d_c$, one can obtain both TM-polarized and TE-polarized GISP modes at the extended range of $\beta$ (see the solid lines). Unlike the SP modes induced at the Bragg gap, these GISP modes are nearly omnidirectional. Moreover, due to the controllable PBGs of the graphene based 1D PCs, one can control the dispersion properties of the GISP modes by adjusting the surface conductivity of the graphene using a gate voltage. To show this, we plotted the dispersion curves of the GISP modes as functions of $\mu_c$ for the TE and the TM polarizations at $\beta = 1.25$ with $d_c = 2d_A$ in Fig. 5. From the figure, it is clear that the frequency of the GISP modes depend on the chemical potential $\mu_c$. So, one can tune the frequency of the GISP mode by applying a suitable external electric field. In the experimental realization, this voltage may be provided by electrodes which are THz transparent dc conductors such as thin doped InSb films [22]. However, these materials show some limitations in the temperature and frequency ranges and the solution of this problem is the subject of new studies.
Since, the GIPBG is solely due to the presence of the graphene monolayers, by replacing the layers $B$ with the layers $A$ the periodic structure still shows the GIPBG [29]. So, a uniform dielectric medium with the periodic graphene multilayers embedded in it may support the GISP modes. To reveal this, we plotted the dispersion curves of the TM-polarized and the TE-polarized GISP modes in the plane of $(f - \beta)$ for $d_c = 0.1d_A$ (the dotted lines), $d_c = d_A$ (the dashed lines) and $d_c = 3d_A$ (the solid lines) by assuming $\varepsilon_B = \varepsilon_A$. The figure indicate that the dispersion properties of the GISP modes do not change by replacing the layers $B$ with the layers $A$ (compare Fig. 4 with Fig. 6). As a result, the existence of the GISP modes is due to the periodic structure of the graphene monolayers.

Finally, we are interested to compare the field structures of the Bragg gap SP modes and the GISP modes. Fig. 7 shows (a) the transverse structure of a GISP mode and (b) the transverse structure of a conventional Bragg gap SP mode having same longitudinal wavenumber $\beta$ with the same geometric parameters. In plotting Fig. 7(a) and (b) we have used $f = 0.925$ THz and $f = 5.175$ THz, respectively, which are the frequencies of the SPs correspond to $\beta = 1.05$. These parameters are chosen from the dispersion curves of the TM-polarized SPs at the first and second PBGs of the graphene based 1D PC for $d_c = 1d_A$ (dashed curves).

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**Fig. 5.** The dispersion curves of the TE-polarized and the TM-polarized GISP modes as functions of $\mu_c$ at $\beta = 1.25$ with $d_c = 2d_A$. Here, the other parameters are the same as Fig. 3.

**Fig. 6.** Same as Fig. 4 with $\varepsilon_B = \varepsilon_A$. 

From Fig. 7(a) one can see that the transverse structure of the GISP mode is strongly localized in the 1D PC and the field does not show the oscillatory behavior inside the structure. This is in contrast to the SPs of the Bragg gap where the fields show some oscillatory behavior inside the 1D PC (see Fig. 7(b)). The strong localization of the SPs at the frequency range of the GIPBG may be due to the special characteristics of this band gap which has been investigated in Ref. [29].

4. Conclusion

Briefly, we studied the existence of the electromagnetic SPs at the interface between a uniform dielectric medium and a 1D PC containing graphene monolayers. It was shown that the structure has a new type of PBG which its characteristic properties depend on the surface conductivity and chemical potential of the graphene sheets. The results show that the structure can support both of the TM-polarized and the TE-polarized SP modes at the frequency range of this new photonic band gap. It is found that the nearly omnidirectional dispersion curves can be obtained for the SPs by adjusting the thickness of the cap layer. In addition, the frequency of the SPs can be controlled by adjusting the chemical potential of the graphene monolayers via external electric field.

References