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Permanently disentangled states of atom–field system via spontaneously generated coherence

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The effect of spontaneously generated coherence on evolution of the entanglement between a driven four-level Y-type atom and its spontaneous emission field is studied. We have shown that the atom will be entangled to its spontaneous emission field due to spontaneously generated coherence and coherent population trapping at the steady state. It is found that the degree of entanglement strongly depends on the initial atomic state. So, it can be controlled by the pumping laser pulses used for preparing an initial atomic system. More interestingly, the atom–field system can be found in a permanently disentangled state for a properly prepared atom.

Keywords: atom–photon; disentanglement; four-level Y-type atom; spontaneously generated coherence

1. Introduction

Quantum entanglement is a quantum mechanical phenomenon in which the quantum states of two or more objects cannot be described by the product of the quantum states of the subsystems [1]. Quantum entanglement has many practical applications in quantum communication [2], cryptography [3], quantum computation [4,5] and has been used to realize quantum teleportation experimentally. For future applications in quantum information science or quantum networks, the faithful mapping of quantum information between a stable quantum memory and a reliable quantum communication channel is essential. Because quantum states can in general not be copied, entanglement between the quantum memory and the communication channel is necessary. Therefore, entanglement between different species like atoms and photons is an essential resource. Combining the advantages of photons (information transport over large distances) and atoms (reliable information storage), atom–photon entanglement enables the interface between atomic quantum memories and photonic quantum communication channels and allows the distribution of quantum information over large distances. It is important for quantum information processing to be able to create entangled states in a controllable way. In recent years, many novel methods have been proposed to generate controllable entangled states [6–9]. Some of them are based on quantum coherence and interference effects [10–14] and using photonic band gap materials [15,16]. There are many ways to generate quantum coherence and interference in an atomic system. The coherence generated in the process of interference of spontaneous emission channels known as spontaneously generated coherence (SGC) [17,18], requires that two close-lying levels be near-degenerate and that the atomic dipole moments be non-orthogonal when the atom is placed in free space. That is, the two close-lying levels should have the same $j$ and $m_j$ quantum numbers [19]. But these rigorous conditions are rarely met in real atoms. Recently, some other methods have been discussed to generate the quantum interference effects in more effective manners such as use of microwave field [20,21]. The SGC has been intensively studied in recent years [22–27]. This effect is responsible for many important physical phenomena, which are potentially applied in optical bistability [28], lasing without population inversion [29–31], coherent population trapping (CPT) [32–34], group velocity reduction [35,36], ultrafast all-optical switching [37], transparent high-index materials [38–40], modified quantum beats [41], dark-state polaritons [42], etc.

In this paper we concentrated on the role of SGC on the entanglement between a coherently driven four-level Y-type atom and its spontaneous emission field. Among the four-level schemes, the Y-type schemes have attracted a lot of attention [43–48]. At present, the Y-type model with different field configurations has not yet been studied for the purpose of controlling the atom–photon entanglement. Although the evolution of the entanglement between the atom and its spontaneous emission field have been investigated previously [49–52], however these works are limited to the case of three-level A-type or V-type atomic systems. In the references [49–51], the authors showed that

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the entanglement of the Λ-type system, which is initially zero, quickly rises to its maximum at early times, then slowly declines to a fixed value at the steady state. They revealed that the steady state entanglement of the system can be controlled by properly adjusting the atomic coherency, the Rabi frequency and the phase of the coupling fields. In the reference [52] the authors showed that the steady state entanglement of the V-type three-level atom and its spontaneous emission field can be controlled by intensity or relative phase of applied fields only in the presence of complete quantum interference. Since, the complete quantum interference requires the transitions from two close-lying levels with parallel dipole moments we use a microwave field to drive no close-lying upper levels of the atomic system. The proposed scheme is therefore independent of the alignment of the dipole moments. However, it requires three driving fields. Since, the microwave source is more readily available and the control of a microwave driving field is easier in comparison with an extra laser field, it is more convenient in its experimental realization. We show that the atom and its spontaneous emission field will be in a disentangled state at the steady-state in the absence of SGC, while, depending on the atomic parameters, they can be entangled at the steady-state in the presence of SGC. More interestingly, the atom–field system can be found in a time-independent disentangled state in the case of the complete CPT. The organization of this paper is as follows. In Section 2, we present the model and the basic theory to investigate the entanglement. Section 3 is devoted to the entanglement analysis of the system. In Section 4, we present a summary of our main results.

2. Theory of atom–photon entanglement

Consider a four-level Y-type atomic system coupled by two laser fields and a microwave driving field, as shown in Figure 1(a). The transition |1⟩ ↔ |2⟩, of frequency ω21 is driven by a coherent coupling laser with Rabi frequency Ω1 and angular frequency ωλ1. A microwave field with frequency ωm2 and Rabi frequency Ω2 drives the magnetic dipole transition between the upper levels |2⟩ and |3⟩ with the hyperfine splitting frequency ω32. Another coherent laser with Rabi frequency Ω3 and angular frequency ωλ3 is applied to the transition |1⟩ ↔ |3⟩ of frequency ω31. We assume that the transition from level |1⟩ to ground level |4⟩ is coupled by the vacuum modes in the free space. We set Ω1 and Ω3 as real parameters, but Ω2 as a complex parameter: Ω2 = |Ω2| exp(iΦ2).

The Hamiltonian describing the dynamics of the system in the interaction picture and the rotating wave approximation with the assumption of h = 1, takes the form:

\[ H = Ω_1 \exp(i Δ_1 t)|2\rangle\langle 1| + Ω_2 \exp(i Δ_2 t)|3\rangle\langle 2| + \sum_k g_k \exp(i δ_k t)|1\rangle\langle 4|b_k + H.C. \]  

(1)

\[ \frac{d}{dt} |Ψ_{AF}(t)\rangle = -i H |Ψ_{AF}(t)\rangle, \]  

(3)

and use the well-known Weisskopf–Wigner theory [55], to achieve the following equations for the atomic probability amplitudes:

\[ \hat{A}(t) = -i M \hat{A}, \]  

(4)

\[ \hat{a}_k(t) = -ig_k \exp(-i δ_k t)A(t), \]  

(5)

with

\[ M = \begin{bmatrix} a_1(t) & a_2(t) & a_3(t) \\ a_2^*(t) & a_1(t) & a_3(t) \\ a_3^*(t) & a_2^*(t) & a_1(t) \end{bmatrix}, \]  

\[ A = \begin{bmatrix} -\frac{i}{2} \Omega_1 & \Omega_3 & 0 \\ 0 & -\frac{i}{2} \Omega_2 & \Omega_1 e^{-i Δt} \\ 0 & 0 & -\frac{i}{2} \Omega_3 e^{-i Δt} \end{bmatrix}, \]  

\[ Δ = Δ_1 + Δ_2 - Δ_3, γ = 2π |g_k|^2 D(ω_k) \]  

is the spontaneous decay rate from level |1⟩ to level |4⟩, and \( D(ω_k) \) is the vacuum mode density at frequency \( ω_k \) in the free space.
By solving Equations (4) and (5) and obtaining the combined atom–field density operator \( \rho_{AF}(t) = |\Psi_{AF}(t)\rangle \langle \Psi_{AF}(t)| \), we get the reduced atomic density operator \( \rho_{A}(t) \) and the reduced spontaneous emission field density operator \( \rho_{F}(t) \) as:

\[
\rho_{A}(t) = Tr_{F}\{\rho_{AF}(t)\}, \quad \rho_{F}(t) = Tr_{A}\{\rho_{AF}(t)\}. \tag{6}
\]

The reduced atomic and field density operators can be used to define the quantum entropies of the atom and the spontaneous emission field. For a bi-component system initially in a pure state, the quantum entropy is a very accurate measure of the degree of the entanglement between two components [56,57]. The higher the reduced quantum entropy, the greater the entanglement. Since, the atom and its spontaneous emission field is a bi-components quantum system, we can use the quantum entropy as a measure of the degree of the entanglement between the atom and its spontaneous emission field. The entropies of the atom and the spontaneous emission field are defined through their respective reduced density operators as

\[
S_{i}(t) = -Tr_{i}\{\rho_{i}(t) \log \rho_{i}(t)\}, \quad (i = A, F). \tag{7}
\]

As presented by Araki and Lieb [58], \( S_{A}(t) \) and \( S_{F}(t) \) are linked to the total entropy of the atom–field system

\[
(S_{AF}(t) = -Tr_{AF}\{\rho_{AF}(t) \log \rho_{AF}(t)\}) \quad \text{as} \quad |S_{A}(t) - S_{F}(t)| \leq S_{AF}(t) \leq S_{A}(t) + S_{F}(t). \tag{8}
\]

Since, \( \rho_{AF}(t) \) is governed by a unitary time evolution, consequently the total entropy of the atom–spontaneous emission field system is time independent. Due to the fact that the atom and the vacuum are initially in a disentangled
For different \( \theta_0 \) at \( \tan \alpha_0 = -\Omega_1/\Omega_2 \). Here, \( \Delta_1 = 0.25, \Delta_3 = 0.75, \Omega_1 = 1.5, \Omega_2 = (\Delta_1 \Delta_3)^{1/2}, \Omega_3 = \Omega_1 (\Delta_1 / \Delta_1)^{1/2} \) and \( \gamma = 1 \). (The color version of this figure is included in the online version of the journal.)

As a result, the populations of the excited states of the atom of the journal.)

The color version of this figure is included in the online version.

Figure 5. The evolution of the entanglement as a function of \( \gamma t \) for

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pure state, the total entropy of the atom–field system is zero. As a result, \( S_A(t) = S_{AF}(t) \). Accordingly, the quantum entropy of the four-level Y-type atom can be expressed as:

\[
S_A(t) = - \sum_{i=1}^{3} |a_i(t)|^2 \log |a_i(t)|^2 - \left[ 1 - \sum_{i=1}^{3} |a_i(t)|^2 \right] \log \left[ 1 - \sum_{i=1}^{3} |a_i(t)|^2 \right].
\] (9)

In what follows, we numerically investigated the entanglement between a coherently driven four-level Y-type atom and its spontaneous emission field using Equation (9).

3. Results and discussion

First of all, we consider the time evolution of the entanglement between the atom and its spontaneous emission field. Figure 2 shows the evolution of the entanglement between the atom and its spontaneous emission field as a function of scaled time \( \gamma t \). Here, the atom–field system was assumed to be initially prepared in the non-entangled state \( |\Psi_{AF}(0)\rangle = 2^{-1/2}(|2\rangle + |3\rangle)|0\rangle \) (where \( \theta_0 = 0 \) and \( \alpha_0 = \pi/4 \)). As the figure demonstrates, due to the population transfer from the excited states to the ground state \( |4\rangle \), the initially zero entanglement between the atom–field system reaches its maximum value and then goes to zero at the steady state. As a result, the populations of the excited states of the atom become zero at the steady state (see Figure 2(b)). Hence, the state of atom–field system at the steady state is given by the non-entangled state \( |\Psi_{AF}(t \to \infty)\rangle = 4 \sum_k a_k(t \to \infty)|1_k\rangle \). The long time probability amplitude \( a_k(t \to \infty) \) can be calculated from Equations (4) and (5) by using the final value theorem and the Laplace transform method. It is not difficult to show that the spontaneous emission spectrum of the system \( S(\delta k) \) is proportional to \( |a_k(t \to \infty)|^2 \) [59] which is plotted in Figure 2(c) as a function of \( \delta k \).

Inspection of Equation (4) reveals that the steady state populations of the excited states are generally zero. However, the populations may be coherently trapped in the upper levels \( |2\rangle \) and \( |3\rangle \) due to SGC. For non-vanishing steady-state population in the upper states of the system, the determinant of the coefficient matrix \( M \) in Equation (4) should be equal to zero. We can easily show that in our system the CPT can be realized under the following conditions:

\[
\Delta = 0 \quad (\Delta_2 = \Delta_3 + 1), \quad \Delta_1 \Delta_3 \geq 0,
\]

\[
\Delta_1 \Omega_3^2 = \Delta_3 \Omega_1^2, \quad \Omega_2 = (\Delta_1 \Delta_3)^{1/2} \exp(i\Phi_2),
\]

\[
\Phi_2 = \begin{cases} 0 & \text{for } \Delta_1 \geq 0, \Delta_3 \geq 0, \\ \pi & \text{for } \Delta_1 < 0, \Delta_3 < 0. \end{cases}
\] (10)

Now, we consider the evolution of the entanglement between the atom and its spontaneous emission field in such a case. Figure 3(a) shows the evolution of the entanglement as a function of scaled time \( \gamma t \) under the CPT conditions. Here, the initial state of the atom–field system was assumed to be the non-entangled state \( |\Psi_{AF}(0)\rangle = |2\rangle|0\rangle \) (where \( \theta_0 = 0 \) and \( \alpha_0 = \pi/2 \)). From Figure 3(a), we see that the atom–field system will be in an entangled state at the steady state. The long time state of the atom–field system can be obtained by knowing the long time probability amplitudes \( a_i(t \to \infty) \) \( (i = 1, 2, 3) \), and \( a_k(t \to \infty) \). It is straightforward to show that the long time probability amplitudes \( a_i(t \to \infty) \) \( (i = 1, 2, 3) \) are given by

\[
a_1(t \to \infty) = 0,
\]

\[
a_2(t \to \infty) = \frac{\cos \theta_0 \cos \alpha_0}{1 + (\Omega_1/\Omega_2)^2} \left( \tan \alpha_0 - \Omega_1/\Omega_3 \right),
\]

\[
a_3(t \to \infty) = -\Omega_1/\Omega_3 a_2(t \to \infty),
\] (13)

under the CPT conditions. For the chosen parameters in Figure 3, we have \( a_2(t \to \infty) = 0.75 \) and \( a_3(t \to \infty) = -0.433 \). Figure 3(b) which shows the time evolution of the populations of the atomic states under the CPT conditions, confirms our results. In this case, the long time state of the atom–field system is \( |\Psi_{AF}(t \to \infty)\rangle = (0.75|2\rangle - 0.433|3\rangle)|0\rangle + |4\rangle \sum_k a_k(t \to \infty)|1_k\rangle \). As before, \( a_k(t \to \infty) \) may be deduced from the spontaneous emission spectrum (see Figure 3(c)).

From Equations (11)–(13) we see that the steady-state probability amplitudes \( a_2(t \to \infty), a_3(t \to \infty) \), and hence the steady-state entanglement of the atom–field system strongly depends on the initial state of the atom. To show this, the steady-state entanglement between the atom–field system is depicted as a function of \( \alpha_0 \) for different \( \theta_0 \) in Figure 4. Despite the CPT conditions, we see that the steady-state entanglement of the system will be zero in two cases. One of these cases is independent from the initial population of the level \( |1\rangle \) (or \( \theta_0 \)) and occurs at \( \tan \alpha_0 = \Omega_1/\Omega_2 \) which corresponds to \( a_2(t \to \infty) = a_3(t \to \infty) = 0 \) (see Equations (12) and (13)). So, the atom–field system will
be in the disentangled state $|\Psi_A F(t \rightarrow \infty)\rangle = |4\rangle \sum_k a_k (t \rightarrow \infty)|1_k\rangle$ at the steady state. The other zero steady-state entanglement occurs at $\theta_0 = 0$ and $\tan \alpha_0 = -\Omega_3/\Omega_1$. Using Equations (12) and (13) we can easily show that the populations of the excited states which are given by $a_1(t) = 0$, $a_2(t) = \sin \alpha_0$ and $a_3(t) = \cos \alpha_0$ are time-independent. Consequently, the population completely is trapped on the upper levels [2] and [3]. Accordingly, the state of the atom–field system is all the time disentangled state $|\Psi_A F(t)\rangle = (\sin \alpha_0|2\rangle + \cos \alpha_0|3\rangle)|0\rangle$. This situation corresponds to the complete quenching of the spontaneous emission spectrum (see Equation (4)). The complete quenching of the spontaneous emission and complete population trapping can be understood in the dressed state picture. By diagonalizing Hamiltonian (1) for the states $|1\rangle$, $|2\rangle$, $|3\rangle$ and the driving fields $\Omega_1, \Omega_2, \Omega_3$, we get three dressed states $|-, 0\rangle$, $|+, 0\rangle$ as shown in Figure 1(b). Therefore, the spontaneous emission from $|1\rangle$ to $|4\rangle$ can be described in terms of the dressed states $|-, 0\rangle$, $|0\rangle$, $|+, 0\rangle$ decaying to level $|4\rangle$ [45]. Under the CPT conditions, quantum interference among these competitive decay channels depending on the initial atomic state may be partially or completely destructive. This partial or complete destructive quantum interference leads to partial or complete population trapping and partial or complete fluorescence quenching. Accordingly, the atom–field entanglement under the CPT conditions strongly depend on the initial atomic state. To show the effect of the initial atomic state on the time evolution of the entanglement of the system under the CPT condition, we plot the entanglement as a function of scaled time $\gamma t$ for different $\theta_0$ at $\tan \alpha_0 = -\Omega_3/\Omega_1$ in Figure 5. The figure indicates that the entanglement between the atom and its spontaneous emission field can be controlled by proper choice of the initial atomic state under the CPT conditions.

As a real Y-type four-level system, we can mention the Rb atom with $5S_{1/2}$, $5P_{3/2}$, $5D_{3/2}$, and $5D_{5/2}$ as the states $|4\rangle$, $|1\rangle$, $|2\rangle$ and $|3\rangle$, respectively. Here, the state $5D_{3/2}$ is coupled to the state $5D_{5/2}$ by a resonant microwave field with frequency around 120 GHz. The value of the spontaneous decay rate $\gamma$ in this system is 6 MHz.

4. Conclusion

In summary, we have shown that the degree of entanglement between a driven four-level Y-type atom and its spontaneous emission field strongly depends on the initial atomic state under coherent population trapping conditions and can be controlled by an appropriate choice of the initial atomic state. Specifically, the atom–field system can be found in a permanently disentangled state due to the spontaneously generated coherence and complete coherent population trapping. In the dressed-state picture, complete destructive quantum interference between the decay channels leads to the complete population trapping and all the time disentanglement of the atom from its spontaneous emission field.

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