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New method for computation of band structures in 1D photonic crystals based on the Fresnel equations

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In this paper, we present a new method for calculation of band structure in one-dimensional bilayer photonic crystals, based on the Fresnel equations. We derive a new relation to obtain the band structure without using the Floquet theorem. It is shown that this relation can be simplified under the assumption that the single-path phase-shift acquired through the individual layers of the photonic crystal be equal to $\pi/2$. The results obtained by our method are compared with the ones obtained from the transfer matrix method to show that they are exactly identical.

Keywords: bilayer photonic crystals; band structure; Floquet theorem; Fresnel equations

1. Introduction

After introducing the concept of photonic crystals (PCs) by Yablonovitch [1] and John [2], much attention has been paid both theoretically and experimentally to optical properties of PCs [3–5]. The PC structure is an artificial dielectric or metal–dielectric structure where its refractive index has modulated at the scale of optical wavelength. The periodic modulation of refractive index gives rise to the formation of the forbidden photonic band gap (PBG), i.e. the range of frequencies for which electromagnetic waves are restricted to propagate. Due to this property, the PCs can offer the capability of manipulating light [6], producing an invisible cloak [7] and potential commercial applications [8]. Measurements and analysis of PBGs are key issues in research and application of PCs [9]. A variety of numerical methods have been developed to demonstrate the nature of photons propagating in a PC structure. In an early attempt, Brillouin studied wave propagation in multi-dimensional periodic media [10], Yariv and Yeh described light propagation in a periodic layered stack of two alternating simple isotropic dielectrics through matrix approaches [5,11]. As the most popular approaches used to describe the light propagation and the PBG structure of PCs, the plane wave (PW) method [3,12], the finite element (FE) method [13], the finite different time domain (FDTD) method [14,15], and the transfer matrix (TM) method [16,17] can be mentioned. With respect to these methods, the PW method was the first one which was used to deal approximately with photonic structures. However, the PW method is subject to the computation of eigenvalues of large matrices and has poor convergence properties. Hence, the results have not been strictly verified by experiments, not even for the simplest case of one-dimensional (1D) PC structures. In contrast, the FE method is capable of dealing with such problems in multi-dimensions, but using sparse matrices. Therefore, the efficiency and the accuracy are both improved. On the other hand, the FDTD method, in principle, can be applied to a photonic crystal, but may not be accurate enough to make a reliable predication of the optical properties to satisfy the complicated and continuous boundary condition at the complex internal interface [18]. The experimental results show good agreement with the results of the TM method. So, the TM method is suitable for the design of 1D PC devices with high precision, but is limited when dealing with 2D and 3D photonic structures due to the inability to solve the Maxwell equations strictly at an interface with complicated boundary conditions [19].

In this work, we introduce a new method to describe the PBG structure of the 1D PCs based on the Fresnel equations. Unlike the TM method which relies on the Floquet theorem, we do not make any assumption about the periodicity of the system and its consequences. It is shown that the results obtained by our method are the same as the results obtained with the TM method. The plan of this paper is as follows: in Section 2, we present the method of calculation employed here. Section 3 is devoted to the presentation of the numerical results. The conclusions are given in Section 4.

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2. Theoretical model and calculations

Consider an infinite 1D PC with a bilayer unit cell composed of two isotropic, and homogeneous layers A and B. The thicknesses and the relative permittivity and permeability of the layers A, B are denoted by \(d_A, d_B, \varepsilon_A, \mu_A\) and \(\varepsilon_B, \mu_B\), respectively. We assume that the reflection coefficient of the PC structure for a light beam incident from a medium refraction at the interface of the surrounding medium and an individual slab layer with the reflection and refraction coefficients \(r_i, \rho_i\). (The color version of this figure is included in the online version of the journal.)

By applying the relations \(|r_i|^2 + |\rho_i|^2 = 1\) and \(\phi_{r_i} + \phi_{\rho_i} = 2\phi_{r_i} + \pi\), Equation (4) is written as:

\[
|\cos(\phi_{r_i})| \geq |t_i|.
\]

Here, \(r_i, t_i (i = A, B)\) are the reflection and transmission coefficients of the slab layer \(i\). Using Equation (6), Inequality (5) may be written in the following form:

\[
||r_A|| - \cos(\phi_A) \geq |t_A| |t_B|.
\]

The reflection and transmission coefficients of a slab layer \(i\) with the relative permittivity \(\varepsilon_i\) and permeability \(\mu_i\) can be obtained in terms of the reflection coefficient \(r_i\) at the interface of the surrounding medium and the slab layer (see Figure 2(b)) as:

\[
r_i = \rho_i (1 - \rho_i^2) \exp(2i\Delta_i),
\]

\[
t_i = \frac{(1 - \rho_i^2) \exp(2i\Delta_i)}{1 - \rho_i^2 \exp(2i\Delta_i)},
\]

where \(\Delta_i = (\omega/c)\sqrt{\varepsilon_i\mu_i - \varepsilon_0\mu_0 \sin^2(\theta)}\) is the single-path phase-shift acquired through a slab of thickness \(d\). Here, \(\omega\) is the frequency, \(c\) is the light speed and \(\theta\) is the incident angle. The absolute values and the phases of \(r_i\) and
\[ t_i \text{ are given by:} \]
\[
|t_i| = \frac{2\rho_i \sin(\Delta_i)}{\left[\rho_i^4 - 2\rho_i^2 \cos(2\Delta_i) + 1\right]^{1/2}}, \\
\tan(\phi_{t_i}) = \frac{1 - \rho_i^2}{(1 + \rho_i^2) \tan(\Delta_i)}, \quad (9)
\]

Substituting Equations (9) in Inequality (7) yields
\[
\Gamma'(\theta, \omega) \leq 0 \quad \text{or} \quad \Gamma'(\theta, \omega) \geq 1. \quad (10)
\]

This is our main result which gives the necessary and sufficient condition to have PBG in the infinite 1D PC. Here,
\[
\Gamma'(\theta, \omega) = K(\theta)\sin(\Delta_A)\sin(\Delta_B) + \sin^2[(\Delta_A + \Delta_B)/2], \quad (11)
\]
with
\[
K(\theta) = (\rho_A - \rho_B)^2 / [(1 - \rho_A^2)(1 - \rho_B^2)]. \quad (12)
\]

Using the Fresnel equations, the reflection coefficient \(\rho_i\) \((i = A, B)\) at the interface of the surrounding medium and the slab layer \(i\) can be expressed as:
\[
\rho_i = \frac{\epsilon_i^{1/2}\mu_0^{1/2}\cos(\theta)/\alpha_0 - \epsilon_i^{1/2}\mu_0^{1/2}\cos(\theta')/\alpha_i}{\epsilon_i^{1/2}\mu_0^{1/2}\cos(\theta)/\alpha_0 + \epsilon_i^{1/2}\mu_0^{1/2}\cos(\theta')/\alpha_i}. \quad (13)
\]

Here, \(\theta'\) is the refraction angle which is related to the incident angle \(\theta_i\) according to Snell’s law as \(\cos(\theta') = \epsilon_i^{1/2}\mu_0^{1/2}\sin(\theta) = \epsilon_i^{1/2}\mu_0^{1/2}\sin(\theta_i)\) and
\[
\begin{cases}
\alpha_0 = \mu_0, \quad \alpha_i = \mu_i, & \text{for TE-polarization,} \\
\alpha_0 = \epsilon_0, \quad \alpha_i = \epsilon_i, & \text{for TM-polarization.} \quad (14)
\end{cases}
\]

In the case where the single-path phase-shift acquired through the slab \(\Delta_i\) \((i = A, B)\) is equal to \(\pi/2\), the inequality (10) reduces to
\[
K(\theta) \leq -1 \quad \text{or} \quad K(\theta) \geq 0. \quad (15)
\]

In order to have \(\Delta_i = \pi/2\) at an arbitrary incident angle \(\theta = \theta_0\) for a given frequency \(\omega = \omega_0\), the thicknesses of the layers must be chosen properly, namely
\[
d_i = \frac{c\pi}{2\omega_0 \left[\epsilon_i\mu_i - \epsilon_0\mu_0 \sin^2(\theta_0)\right]^{1/2}}, \quad (i = A, B). \quad (16)
\]

For this choice of \(d_i\), the phase-shift \(\Delta_i\) varies with \(\theta\) symmetrically around \(\pi/2\) as:
\[
\Delta_i = \frac{\pi}{2} \left(\frac{\epsilon_i\mu_i - \epsilon_0\mu_0 \sin^2(\theta_0)}{\epsilon_i\mu_i - \epsilon_0\mu_0 \sin^2(\theta_0)}\right)^{1/2}, \quad (17)
\]
and it will be approximately equal to \(\pi/2\) for larger values of \(\epsilon_i\mu_i\). It must be mentioned that, in deriving our results for the PBG condition, we have neglected the effect of losses.

3. Results and discussion

In what follows we want to use Inequality (10) to obtain the photonic band structures of 1D PCs. In order to validate our method, we compare our results with the results obtained by the TM method. As a first example, we consider a 1D PC composed of ordinary dielectric materials with \(n_A = 2.22, \quad d_A = 300\, \text{nm}, \quad n_B = 1.46\) and \(d_B = 150\, \text{nm}\). Initially, we examine the behavior of \(\Gamma(\bar{\beta}, \tilde{\omega})\) as a function of \(\beta\) at the given frequency \(\tilde{\omega}\). Here, \(\tilde{\omega} = \omega/\Lambda/2\pi\) is the reduced frequency and \(\bar{\beta} = \beta\Lambda/2\pi\) represents the reduced parallel component of the wave vector \(\beta\). The parameters used are \(n_A = 2.22, \quad d_A = 300\, \text{nm}, \quad n_B = 1.46\) and \(d_B = 150\, \text{nm}\) (same as the parameters used in [21,22]). (The color version of this figure is included in the online version of the journal.)

Figure 3. Plot of \(\Gamma(\bar{\beta}, \tilde{\omega})\) versus \(\bar{\beta}\) at (a) \(\tilde{\omega} = 0.7\) and (b) \(\tilde{\omega} = 1.25\) for TE-polarized (solid lines) and TM-polarized (dashed lines) waves. Here, the shaded region shows the allowed reduced parallel component of the wave vector \(\bar{\beta}\). The parameters used are \(n_A = 2.22, \quad d_A = 300\, \text{nm}, \quad n_B = 1.46\) and \(d_B = 150\, \text{nm}\) (same as the parameters used in [21,22]). (The color version of this figure is included in the online version of the journal.)

As a second example, we consider a 1D PC consisting of two kinds of different single negative materials with the parameters \(\epsilon_A = 3, \quad \mu_A = 1 - 100/\omega^2, \quad d_A = 10\, \text{mm}, \quad \epsilon_B = 1 - 144/\omega^2, \quad \mu_B = 1.0\) and \(d_B = 5\, \text{mm}, \) respectively.
Here, we want to obtain the photonic band structure of the system by our formulism and compare the results with the results of [23]. We first plot $\Gamma(\theta, \omega)$ as a function of incident angle $\theta$ for both TE (solid lines) and TM (dashed lines) polarizations at the frequencies (a) $\omega = 11.5$ GHz and (b) $\omega = 43$ GHz in Figure 6. Here, the shaded region shows the allowed incident angles $\theta$ at the given frequencies. From
the figure one can obtain the allowed or forbidden incident angles \( \theta \) at the given frequency \( \omega \). As well, the plot of \( \Gamma(\theta, \omega) \) as a function of frequency \( \omega \) for both TE (solid lines) and TM (dashed lines) polarizations at the incident angles (a) \( \theta = 15^\circ \) and (b) \( \theta = 45^\circ \) are shown in Figure 7. Here, the shaded region shows allowed frequency \( \omega \) at the given incident angles. The figure shows some frequency ranges in the forbidden band, independent from the incident angle. To see this better, we apply Inequality (10) to obtain the band structure of the 1D PC on the plane of \( \theta - \omega \). Figure 8 shows the photonic band structure of the system obtained by our formulism for both TE and TM polarizations. Here, the unshaded regions show the PBG of the 1D PC structure. As one can see from Figure 8, the band structure of the 1D PC contains some omnidirectional band gaps at the single-negative frequency region. The properties of these new types of omnidirectional gaps have been investigated in [23] using the TM method. Comparing Figure 8 with Figure 1(b) in [23] reveals that the photonic band structures obtained by our method are the same as the results of the TM method.

In the remaining part of the paper, we investigate the photonic band structure of a 1D PC composed of dielec-
tric layers of polydiacetylene 9-BCMU organic material with linear refractive index \( n_A = 1.55 \) and TiO\(_2\) material with refractive index \( n_B = 2.3 \) [24] whose thicknesses are selected according to Equation (16). Here, we consider two structures with different thicknesses of the layers A and B. We assume that \( \omega_0 = 3.2725 \times 10^{15} \text{ rad s}^{-1} \) (corresponds to \( \lambda_0 = 576 \text{ nm} \)) and choose \( d_A, d_B \) according to Equation (16) with (a) \( \theta_0 = 0^\circ \) and (b) \( \theta_0 = 53^\circ \). Then, in the first structure the thicknesses of the layers will be \( d_1 = 93 \text{ nm}, d_2 = 62 \text{ nm} \), while in the second structure we have \( d_1 = 108 \text{ nm}, d_2 = 66 \text{ nm} \). The plot of \( \sin(\Delta_1), \sin(\Delta_2) \) and \( \sin[(\Delta_1 + \Delta_2)/2] \) as functions of incident angle \( \theta \) are shown in Figure 9. The figure indicates that the condition \( \Delta_i \equiv \pi/2 \) \( (i = A, B) \) is realized at all incident angles for both cases of (a) \( \theta_0 = 0^\circ \) and (b) \( \theta_0 = 53^\circ \). Now, we plot \( K(\theta) \) versus \( \theta \) in Figure 10. From Figure 10 it is clear that \( K(\theta) > 0 \) for all incident angles. So, we may have a forbidden band at the frequency \( \omega = \omega_0 \). The angular properties of this forbidden band can be verified using Equation (11). Figure 11 shows the plot of \( \Gamma(\theta, \omega_0) \) as functions of \( \theta \) for both structures. As Figure 11 reveals, one can not achieve an omnidirectional relectivity using the first structure with \( d_1 = 93 \text{ nm}, d_2 = 62 \text{ nm} \) at the frequency \( \omega_0 \). Whereas, the second PC with \( d_1 = 108 \text{ nm}, d_2 = 66 \text{ nm} \) has an omnidirectional reflectivity at the frequency \( \omega_0 \). In order to see these properties clearly, we use the TM method to plot the reflectivity of a finite PC with the structure \((AB)^2\) [25] for both cases at the frequency \( \omega = \omega_0 \) as depicted in Figure 12. One can easily see that Figure 12 confirms the results of Figure 11. In continuation, we use Inequality (10) to obtain the band structure of these 1D PCs around the frequency \( \omega_0 \). Figure 13 shows the band structure of the systems on the plane of \( \omega-\theta \). This figure reveals that one can obtain an omnidirectional PBG with the 1D PC composed of ordinary dielectric materials by proper choice of the thicknesses of the layers \( (d_A, d_B) \) (see Figure 13(b)). These results are exactly the same as the results obtained by the TM method which is plotted in Figure 14 for the structure \((AB)^2\) [25].

4. Conclusion

We presented here the application of the Fresnel equations to obtain the band structure of one-dimensional periodic media without using the Floquet theorem. The condition governed on the photonic band structure was greatly simplified by the assumption of a \( \pi/2 \) phase-shift through the individual layers of the photonic crystals. We calculated the band structures for both TE-polarized and TM-polarized waves on the plane of frequency and incidence angle. It was shown that our results are exactly the same as ones obtained using the TM method. Finally, several examples have been presented to show the applicability of the approach.

References