Position Dependent Spontaneous Emission Spectra of a Λ-Type Atomic System Embedded in a Defective Photonic Crystal

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Abstract We investigate the position dependent spontaneous emission spectra of a Λ-type three-level atom with one transition coupled to the free vacuum reservoir and the other one coupled to a double-band photonic band gap reservoir with a defect mode in the band gap. It is shown that, for the atom at the defect location, we have a two-peak spectrum with a wide dark line due to the strong coupling between the atom and the defect mode. While, when the atom is far from the defect location (or in the absence of the defect mode), the spectrum has three peaks with two dark lines due to the coupling between the atom and the photonic band gap reservoir with the largest density of states near the band edges. On the other hand, we have a four-peak spectrum for the atom at the space in between. Moreover, the average spontaneous emission spectra of the atoms uniformly embedded in high dielectric or low dielectric regions are described. It is shown that the atoms embedded in high (low) dielectric regions far from the defect location, effectively couple to the modes of the lower (upper) photonic band. However, the atoms embedded in high dielectric or low dielectric regions at the defect location, are coupled mainly to the defect modes. While, the atoms uniformly embedded in high (low) dielectric regions with a normal distance from the defect location, are coupled to both of defect and lower (upper) photonic band modes.

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1 Introduction

The modification and control of spontaneous emission spectrum is an active and widely discussed research topic due to the potential applications such as lasing without inversion, high-precision spectroscopy, quantum information and computing. Since, the spontaneous emission spectrum depends on the energy structure of the atoms and the nature of surrounding environment, the spontaneous emission spectrum of the atoms embedded in the photonic band gap (PBG) materials have been studied extensively in recent years. From the point of view of the surrounding environment of the atoms, the density of states (DOS) of PBG structures have significant deviation from that of free space vacuum. The main property of the PBG materials is the existence of an absolute band gap, a frequency region over which there can be no propagation of light in the structure whatever the direction of propagation. The most interesting phenomena in the interaction of the atoms with the PBG structures arise from the rapid variations of the photon DOS of these materials. The study of quantum and nonlinear optical phenomena, in atoms embedded in the PBG structures, leads to the prediction of many interesting effects. As examples, the localization of light and the formation of photon-atom bound states, suppression and even complete cancellation of spontaneous emission, population trapping in two-atom systems, the phase dependent behavior of the population dynamics and other phenomena can be mentioned. Recently, the spontaneous emission spectra of a three-level atom embedded in a double-band PBG structure have been studied by Cheng et al. Unlike the other studies, the authors assumed that the atoms embedded in a photonic crystal (PC) interact with a two-band coherent PBG reservoirs. Regarding Refs. [39–40], they revealed that the atom-field coupling strengths depend on the atomic position. As a result, the spontaneous emission spectra of the atoms embedded in high dielectric regions are blue-shifted while those of the atoms in low dielectric regions are red-shifted. By adding extra dielectric material where it does not belong or by removing some of the dielectric material that should be there, defects may be introduced in a PC. The presence of defects (local deviations from the ideal crystalline structure) in the PC may drastically change the optical properties of the PBG material. A defect destroys the perfect translational symmetry of the PBG material and may lead to the formation of a defect mode within the band gap region, analogous to donor and acceptor defect modes in a doped semiconductor crystal. If a single-defect mode is introduced into the PBG, the density of states of the system will be zero within the band gap, except for isolated peaks associated with the defect mode. The localized mode associated with a defect in an otherwise perfect PBG material...
can act as a microcavity of very high quality factor Q. This high-Q defect mode microcavity can be considered for all the applications in which high-Q optical microcavities are used. It can be used to realize the optical engineers dream of threshold-less lasers.\cite{41-42} Then a simple question may naturally arise: what is the effect of defect modes on the spontaneous emission spectrum of an atom in this coherent PBG reservoir model? To answer this question, we investigate the effect of defect modes on the spontaneous emission spectrum of a Λ-type three-level atom embedded at different positions in an isotropic double-band PBG structure with a defect mode in the band gap. We assume that the upper level of the atom is coupled to one of the lower levels by the free vacuum modes, and to the other lower level by the modes of a defective PBG reservoir. It is shown that the profile of free space spontaneous emission spectrum of the atom depends on the atomic positions. For the atom at the defect location, the spontaneous emission spectrum has two peaks with a wide dark line. When the atom is far from the defect location (or in the absence of the defect mode), the spectrum contains three peaks without any true dark lines. And for the atom at the space in between, it is a four-peak spectrum with three dark lines. This paper is organized as follows: In Sec. 2, we apply the time-dependent Schrödinger equation to describe the dynamics of our system and calculate the spontaneous emission spectra in the free space reservoir. The general calculated results and their analysis are presented in Sec. 3. The major conclusions are summarized in Sec. 4.

2 Equations for Spontaneous Emission Spectrum

Consider a Λ-type three-level atom embedded in a defective PC. The atom consists of an upper level \( |2\rangle \) and two lower levels \(|1\rangle, |0\rangle\) as shown in Fig. 1. The transition from the upper level |2\rangle to the lower level |1\rangle is coupled by the modes of a double-band isotropic PBG material with a defect mode in the band gap (\( \omega_k \)). While, the transition from the upper level |2\rangle to the lower level |0\rangle is coupled by the free vacuum reservoir (\( \omega_{\lambda} \)). As a physical system containing the configuration shown in Fig. 1, we can mention the Ba\(^+\) ion with the upper level \( |2\rangle = |6p^2P_{1/2}\rangle \) and the lower levels \(|1\rangle = |5d^2D_{3/2}\rangle, |0\rangle = |6p^2S_{1/2}\rangle\), which is doped in a PBG material. Here, \( \omega_{21} = 2.89 \times 10^{15} \text{ rad/s} \) and \( \omega_{20} = 3.82 \times 10^{15} \text{ rad/s} \). A three-dimensional (3D) PC with diamond lattice of air spheres in silicon substrate has a complete band gap at the frequency \( 2.89 \times 10^{15} \text{ rad/s} \) with a gapmidgap ratio of 27.28%, for the lattice constant 382.6 μm and the filling ratio of the air spheres 81%.\cite{43} By perturbing a single lattice site one may cause the appearance of a defect mode in the band gap. Such a system can be used in the experimental observation of the position dependent spontaneous emission.

\begin{equation}
H = \sum_{\lambda} g_{\lambda}^{20} e^{-i\delta_\lambda t} |2\rangle \langle 0| \hat{b}_\lambda + \sum_{\mu} g_{\mu}^{21} e^{-i\delta_\mu t} |2\rangle \langle 1| \hat{b}_\mu + \text{H.C.} \tag{1}
\end{equation}

Here, \( g_{\lambda}^{20} \) is the coupling constant of the atom to the free vacuum mode \( \lambda \), \( \hat{b}_\lambda \) is the annihilation operator for the free vacuum mode \( \lambda \) with detuning \( \delta_\lambda = \omega_\lambda - \omega_{20} \). The index \( \mu \) labels the energy band index \( (n) \) and wave vector \( (k) \) of the defective PBG reservoir, \( \mu = \{n,k\} \). \( \hat{b}_\mu \) is the annihilation operator for the mode \( \mu \) of the defective PBG reservoir with detuning \( \delta_\mu = \omega_\mu - \omega_{21} \). \( g_{\mu}^{21} \) is a position dependent atom-field coupling constant (assumed to be real) between the atomic transition \( |2\rangle \rightarrow |1\rangle \) and the mode \( \mu \) of the defective PBG reservoir which can be given as \( g_{\mu}^{21} = (\omega_{21}/\hbar)[h/2e\lambda_{\omega_\mu}(\lambda_{\mu})^{1/2}]d_{\mu}(r_0) \).\cite{39} Here, \( d_{\mu}(r_0) \) is the atomic dipole moment for the transition \( |2\rangle \rightarrow |1\rangle \) with the atomic transition frequency \( \omega_{21} \). \( E_{\mu}(r_0) \) is the atom-coupling field of the mode \( \mu \). In Ref. [40], Joannopoulos revealed the physical origin of the PBG and found that the modes from the lower band of the PBG (low-frequency modes) concentrate their energy in the regions with high dielectric constant and the modes from the upper band of the PBG (high-frequency modes) concentrate their energy in the low dielectric regions. To illustrate this, we consider the shortest interesting wave, with wavelength 2a (here, a is the lattice constant). One can see that the electric field can be placed in two ways in the crystal without disturbing the symmetry of the unit cell about its center. One way is to place the nodes in the high index material and another with the nodes in the low index material, as shown in Figs. 2(a) and 2(b). The darker material represents the high index material. If the effective dielectric constant (the average of the dielectric constant of where the energy, in Figs. 2(d) and 2(e), is located) for each wave is considered, it becomes obvious that two waves with the same wavelength (same wave vector) will have different
frequencies, and the difference will be bigger for larger refractive index ratio. This means that for the frequencies between the two just mentioned, there will be no allowed relative distance between the atom and the defect location. For an atom embedded near the defect location $\alpha_d(\vec{r}_0) \gg \alpha_l(\vec{r}_0), \alpha_h(\vec{r}_0)$, while for the atom far from the defect location $\alpha_d(\vec{r}_0) \ll \alpha_l(\vec{r}_0), \alpha_h(\vec{r}_0)$. By using these parameters, the position dependent atom-field coupling constant can be approximated by $g_{21}^{n} = g_{k}^{21} \alpha_n(\vec{r}_0)$ ($n = l, d, h$) with real constant

$$g_{k}^{21} \approx \frac{\omega_{21}}{\hbar} \left[ \frac{\hbar}{2\epsilon_0 \omega_k V} \right]^{1/2} d_{21} \cdot \vec{e} E_k.$$ 

We assume that the atom is initially prepared in the upper level $|2\rangle$. Then the state vector of the system at time $t$ can be written as

$$|\psi(t)\rangle = a_2(t)|2\rangle, \{0_\lambda, 0_k\} + \sum_{\lambda} a_\lambda(t)|0\rangle, \{1_\lambda, 0_k\}\}.$$ 

Here $a_2(t)$ is the probability amplitude to find the atom in the upper level $|2\rangle$ with no photons in any vacuum mode. $a_\lambda(t)$ is the probability amplitude to find the atom in the lower level $|0\rangle$ and a single photon in the $\lambda$-th mode of the free vacuum reservoir. Finally, $a_\mu(t)$ is the probability amplitude to find the atom in the lower level $|1\rangle$ and a single photon in the $k$-th mode from the $n$-th band of the defective PBG reservoir. Due to the position dependent electric fields of the PBG modes, the position depending probability amplitude $a_\mu(t)$ can be considered as $a_\mu(t) = a_k(t) \alpha_n(\vec{r}_0)$, So, the probability to find the atom in the lower level $|1\rangle$ can be given by $|a_k(t)|^2$.

By substituting the Hamiltonian (1) and the state vector (2) into the Schrödinger equation, the following probability amplitude equations are obtained:

$$i\dot{a}_2(t) = \sum_{k} g_{2}^{21} \left\{ \alpha_l^2(\vec{r}_0) e^{-i\delta t} + \alpha_d^2(\vec{r}_0) e^{-i\delta t} \right\} a_k(t) + \sum_{\lambda} g_{20}^{20} e^{-i\delta t} a_{\lambda}(t), \quad (3)$$

$$i\dot{a}_{\lambda}(t) = g_{k}^{21} \left\{ \alpha_l^2(\vec{r}_0) e^{i\delta t} + \alpha_d^2(\vec{r}_0) \right\} a_2(t), \quad (4)$$

$$i\dot{a}_k(t) = g_{2}^{21} \left\{ \alpha_l^2(\vec{r}_0) e^{-i\delta t} + \alpha_d^2(\vec{r}_0) \right\} a_2(t). \quad (5)$$

By formal time integration of Eqs. (4)–(5) and eliminating $a_k(t)$ and $a_\lambda(t)$ from Eq. (3) and using the Wigner–eisskopf approximation, we get:

$$\dot{a}_2(t) = -\frac{\epsilon}{2} a_2(t) - \int_0^t G(t-t') a_2(t') dt', \quad (6)$$

where

$$G(t-t') = \alpha_l^4(\vec{r}_0) K_l(t-t') + \alpha_d^4(\vec{r}_0) K_d(t-t') + \alpha_l^2(\vec{r}_0) K_h(t-t') + \alpha_d^2(\vec{r}_0) K_d(t-t')$$

$$+ \alpha_l^2(\vec{r}_0) e^{i\omega_{dt}t} K_l(t-t') + \alpha_d^2(\vec{r}_0) e^{-i\omega_{dt}t} K_l(t-t') + \alpha_l^2(\vec{r}_0) e^{i\omega_{dt}t} K_h(t-t') + \alpha_l^2(\vec{r}_0) e^{-i\omega_{dt}t} K_h(t-t'). \quad (7)$$

Here $\omega_{dt} = \omega_d - \omega_l$, $\omega_{hd} = \omega_h - \omega_d$, $\omega_{ht} = \omega_h - \omega_l$ and

$$K_l(t-t') = \sum_k |g_{k}^{21}|^2 e^{-i(\omega_{l}-\omega_{21})(t-t')} \approx \beta^{3/2} \int d\omega \rho_l(\omega) e^{-i(\omega_{l}-\omega_{21})(t-t')}, \quad (8)$$

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**Fig. 2** (Color online) (a) The electric field profile of lower band modes. (b) The electric field profile of upper band modes. (c) The electric field profile of defect modes. (d) The electric field of lower band modes has its energy concentrated in the high-dielectric region (dark regions). (e) The electric field of upper band modes has its energy concentrated in the low-dielectric region (light regions). (f) The electric field of defect modes has its energy concentrated in the defect location (yellow regions).
\[ K_d(t - t') = \sum_k \tilde{g}_k^2 |^2 e^{-i(\omega_d - \omega_2k)(t-t')} \approx \beta_d^2 \int d\omega \rho_d(\omega) e^{-i(\omega - \omega_2)(t-t')}, \] (9)
\[ K_h(t - t') = \sum_k \tilde{g}_k^2 |^2 e^{-i(\omega_h - \omega_2k)(t-t')} \approx \beta_h^2 \int d\omega \rho_h(\omega) e^{-i(\omega - \omega_2)(t-t')} \] (10)

are memory kernels of the PBG reservoir. The resulting memory kernels depend very strongly on the DOS of the radiation fields of the relevant photon reservoirs. The DOS of the double-band isotropic PBG reservoirs with the defect modes in the band gap are given as\[^{19}\]
\[ \rho_1(\omega) = \frac{c}{2\pi} \sqrt{\omega - \omega_c} \Theta(\omega - \omega_c), \] (11)
\[ \rho_0(\omega) = -\frac{K}{\pi (\omega - \omega_0d)^2 + \gamma_0^2}, \] (12)

where \( \omega_c = \omega_b - \omega_2, \delta_d = \omega_0d - \omega_2, \) and \( \delta_c = \omega_c - \omega_2. \) "erf" is the error function, \( \beta \) and \( \beta_d \) are constants of coupling of the atom to the PBG reservoir and the defect modes, respectively.\[^{22}\]

The aim here is to obtain the long time spontaneous emission spectrum \( S(\delta_\lambda) \) of the atom in the free space transition \( |2\rangle \rightarrow |0\rangle. \) The spontaneous emission spectrum of the atom is the Fourier transform of
\[ \langle E^-(t + \tau)E^+(t) \rangle_{t=\infty} = \langle \psi(t) | \sum_{\lambda, \lambda'} \tilde{b}_\lambda^\dagger \tilde{b}_{\lambda'} e^{i\lambda(t+\tau - t')e^{-i\omega_\lambda t} | \psi(t) \rangle_{t=\infty}. \] (17)

Substituting Eq. (2) into Eq. (17) and performing the sums we get
\[ S(\delta_\lambda) \propto |\tilde{a}_\lambda(t \rightarrow \infty)|^2, \] (18)
Here, \( a_\lambda(t \rightarrow \infty) \) is the long-time probability amplitude to find the atom in the lower level \( |0\rangle \) and a single photon in the \( \lambda\)-th mode of the free vacuum reservoir. By Laplace transform of Eq. (5) and using final value theorem the spontaneous emission spectrum \( S(\delta_\lambda) \) is given by
\[ S(\delta_\lambda) \propto |\tilde{a}_2(s)|^2 = |\tilde{a}_2(s)|^2 \] (19)

where, \( \tilde{a}_2(s) \) is the Laplace transform of the probability amplitude \( a_2(t), \) which can be given as
\[ \tilde{a}_2(s) = \tilde{\zeta}(s) - \{ \alpha^2(\tilde{r}_0)\alpha_\lambda(\tilde{r}_0)\tilde{K}_d(s - i\omega_d)\tilde{\zeta}(s + i\omega_d) + \alpha_h^2(\tilde{r}_0)\tilde{K}_h(s - i\omega_h)\tilde{\zeta}(s + i\omega_h) \]
\[ + \alpha_d^2(\tilde{r}_0)\alpha_\lambda^2(\tilde{r}_0)\tilde{K}_d(s + i\omega_d)\tilde{\zeta}(s + i\omega_d) + \alpha_h^2(\tilde{r}_0)\tilde{K}_h(s + i\omega_h)\tilde{\zeta}(s + i\omega_h) \]
\[ + \alpha_d^2(\tilde{r}_0)\alpha_h^2(\tilde{r}_0)\tilde{K}_d(s + i\omega_d)\tilde{\zeta}(s + i\omega_d) + \alpha_h^2(\tilde{r}_0)\tilde{K}_h(s + i\omega_h)\tilde{\zeta}(s + i\omega_h) \} \tilde{\zeta}(s). \] (20)

Here,
\[ \tilde{\zeta}(s) = \left[ s + \gamma + \alpha^2(\tilde{r}_0)\tilde{K}_d(s) + \alpha_h^2(\tilde{r}_0)\tilde{K}_h(s) + \alpha_d^2(\tilde{r}_0)\tilde{K}_d(s) + \alpha_h^2(\tilde{r}_0)\tilde{K}_h(s) \right]^{-1}, \] (21)
and
\[ \tilde{K}_d(s) = \beta_d^2 \frac{e^{i\pi/4}}{\sqrt{s + i\delta_d + \sqrt{1\gamma_d}}}, \] (22)
\[ \tilde{K}_h(s) = \beta_d^2 \frac{1}{s + i\delta_d + \gamma_d}, \] (23)
\[ \tilde{K}_h(s) = \beta_h^2 \frac{e^{-i\pi/4}}{\sqrt{s + i\delta_h + \sqrt{1\gamma_h}}}, \] (24)

are the Laplace transforms of the memory kernels (14)–(16). In what follow, we assume that the defect modes appear at the middle of the band gap i.e. \( \omega_c - \omega_0d = \omega_0d - \omega_b = \Delta/2 \) where \( \Delta = \omega_c - \omega_b \) is the band gap width. So, \( \delta_c \) and \( \delta_b \) can be expressed in term of \( \delta_d \) as \( \delta_c = \delta_d + \Delta/2 \) and \( \delta_b = \delta_d - \Delta/2. \) In symmetric case in which the atomic transition frequency \( \omega_2k \) is chosen at the middle of band gap, \( \delta_d = \omega_0d - \omega_2 \) is zero, and in the asymmetric case, \( \delta_d \neq 0. \) Since the most of DOS of the PBG are contributed from the states near the band edges, we consider \( \omega_0d \approx 2\omega_d \approx 2\omega_2 \approx \Delta. \)

3 Results and Discussion
Now, we use Eqs. (19)–(24) to investigate the spontaneous emission spectrum of the atom embedded at different positions in the defective PC. Figure 3 shows the
free space spontaneous emission spectra as functions of detuning $\delta_l$ in the symmetric case (the upper panel) and the asymmetric case (the lower panel), respectively. Here, the solid lines show the spectra of the atom embedded far from the defect location with $\alpha_l(\vec{r}_0) = \alpha_h(\vec{r}_0) = 0.70$, $\gamma_d(\vec{r}_0) = 0.09$. The dotted lines show the spectra of the atom near the defect location with $\alpha_l(\vec{r}_0) = \alpha_h(\vec{r}_0) = 0.18$, $\gamma_d(\vec{r}_0) = 0.96$ and the dashed lines show the spectra of the atom at the space in between with $\alpha_l(\vec{r}_0) = \alpha_d(\vec{r}_0) = \alpha_h(\vec{r}_0) = 0.56$. As one can see from Fig. 3 the spontaneous emission spectra of the atom have three peaks with two dark lines at $\delta \lambda = \delta_x$ and $\delta \lambda = \delta_v$ when the atom is far from the defect location or in the absence of the defect modes (see the solid lines). These dark lines are the results of the strong coupling between the atom and the PBG reservoir and the population transfer in the transition $|2\rangle \rightarrow |1\rangle$ near the band edges with the largest density of states. This large density of states is the consequence of isotropic relation, which are used to describe the photon dispersion near the band edge of the PBG material. As one know, there is no physical PBG material with the isotropic photon dispersion relation, and a more realistic picture needs to use the anisotropic photon dispersion relation. Nevertheless, the isotropic model exhibits many features of the atom-photon interaction in three-dimensional PCs. The most significant difference between these models is the existence of the large density of states near the band edges of the isotropic PBG. Due to the mentioned reasons, these dark lines are not true dark lines. However, the spontaneous emission spectra of the atom at the defect location have two peaks with a wide dark line at $\delta \lambda = \delta_d$ (see the dotted lines). This dark line, which is a true dark line is created as a consequence of population transfer in the transition $|2\rangle \rightarrow |1\rangle$ due to the strong coupling between the atom and the defect modes. In this case, we see that the side lobes, which are the result of interaction between the atom and PBG modes, disappear. On the other hand, the spectra of the atom at the space in between have four peaks with three narrow dark lines at $\delta \lambda = \delta_x$, $\delta \lambda = \delta_v$, and $\delta \lambda = \delta_d$ (see the dashed lines). These dark lines are due to the coupling of the atom to both PBG and defect modes. Of course, only the dark line at $\delta \lambda = \delta_d$, which is due to the defect modes is a true dark line.

Now we investigate the effect of the defect modes on the absorption spectra by changing the relevant defect mode parameters. Figure 4 shows the spontaneous emission spectra of the atom embedded at different positions in the defective PC for different $\gamma_d$. Here, the upper panels show the symmetric case and the lower panels show the asymmetric case, for (a) $\gamma_d = 0.01 \gamma$, (b) $\gamma_d = 0.1 \gamma$, and (c) $\gamma_d = 0.3 \gamma$. As we expected, when the atom is far from the defect location (or in the absence of the defect mode), due to the weak interaction between the atom and the defect modes, the free space spontaneous emission spectra are not considerably affected by varying $\gamma_d$ (see the solid lines). On the other hand, for the atom at the defect location, the spontaneous emission spectra are slightly weakened when the parameter is adjusted from $\gamma_d = 0.01 \gamma$ to $\gamma_d = 0.3 \gamma$ (see the dotted lines). Because, for the atom at the defect location, the atom mainly couples to the defect modes and hence the population transfer in the transition $|2\rangle \rightarrow |1\rangle$ increase. For the atom at the space in between, it can be found that the central peaks of the spontaneous emission almost completely quenched when the quality factor $Q = \omega_d/2\gamma_d$ of the defect modes becomes small enough (see the dashed lines).

In the same configuration as we used, Du et al. have investigated the response of an open $\Lambda$-type three-level atom to a probe laser field tuned to the free space transition $|2\rangle \rightarrow |0\rangle$. They revealed that the system can be transparent to a weak probe field in the presence of high-Q defect modes. In this case, the high-Q defect modes can even induce an atom-field coherent population trapping state without involving the excited state of the atom. These effects, which are due to the suppression of spontaneous emission, reveal the importance of dark line in the spontaneous emission spectrum and the significance of the presence of defect modes. Although, to observe the transparency and the coherent population trapping, the atoms must be doped at the defect location in the PC.
Next, we consider the effect of the band gap width $\Delta$ on the spontaneous emission spectra of the atom at different positions. Figure 5 shows the spontaneous emission spectra of the atom as functions of detuning $\delta_\lambda$ for three different values of $\Delta$. Here, the upper panels show the symmetric case and the lower panels show the asymmetric case, for (a) $\Delta = \gamma$, (b) $\Delta = 3\gamma$, and (c) $\Delta = 6\gamma$. For the atom far from the defect location, we see that the side peaks of the spontaneous emission spectrum disappear by increasing the gap width and we get a Lorentzian spectrum (see the solid lines). The Lorentzian spectrum indicates that the atom embedded far from the defect location effectively couples to the free-space vacuum modes rather than the PBG modes. On the other hand, the spontaneous emission spectrum of the atom at the defect location is approximately independent from $\Delta$ (see the dotted lines). However, the spontaneous emission spectrum of the atom at the space in between is affected by the gap width due to the coupling of the atom to both PBG and defect modes. Furthermore, for large gap width, the atom only couples to the free-space vacuum and the defect modes. As a result, we have a two-peak spectrum for large $\Delta$ (see the dashed lines).

In the rest of the paper, we consider the average spontaneous emission spectrum of the atoms uniformly embedded in high (low) dielectric regions at different distance from the defect location. Figure 6 shows the average spontaneous emission spectra of the atoms uniformly embedded (a) in high dielectric regions and (b) in low dielectric regions. Here, we used the solid lines for the atoms far from the defect location, the dotted lines for the atoms at the defect location and the dashed lines for the atoms at the space in between. From Fig. 6 we see that the average spontaneous emission spectra of the atoms uniformly em-
bedded in high (low) dielectric regions far from the defect location have only one dark line (see the solid lines). This dark line which is created at \( \delta = \delta_v \) (\( \delta = \delta_c \)) is due to the singularity of the PBG DOS near the lower (upper) band edge. In this case, the atoms uniformly embedded in high (low) dielectric regions effectively interact with the modes of the lower (upper) band which concentrate their energy in high (low) dielectric regions.

![Image](image.png)

**Fig. 6** (Color online) The average spontaneous emission spectra of the atoms uniformly embedded (a) in high dielectric regions and (b) in low dielectric regions. Here, we used the solid lines for the atoms far from the defect location, the dotted lines for the atoms near the defect location, and the dashed lines for the atoms at the space in between. Also, the upper panels show the symmetric case with \( \delta_d = 0 \), the lower panels show the asymmetric case with \( \delta_d = -0.25\gamma \), respectively. The other parameters are same as Fig. 3.

On the other hand, one can see that the average spontaneous emission spectra of the atoms uniformly embedded in high dielectric or low dielectric regions at the defect location contain a wide dark line at \( \delta = \delta_d \) (see the dotted lines). As mentioned before, this is due to the strong coupling between the atom and defect modes in the transition \( |2 \rangle \rightarrow |1 \rangle \). Furthermore, we see that the average spontaneous emission spectra of the atoms uniformly embedded in high (low) dielectric regions at the space in between have two narrow dark lines at \( \delta = \delta_d \) and \( \delta = \delta_c \) (\( \delta = \delta_v \)). This shows that the atoms uniformly embedded in high (low) dielectric regions with a normal distance from the defect location, are coupled to both of defect and lower (upper) photonic band modes in the transition \( |2 \rangle \rightarrow |1 \rangle \) (see the dashed lines).

### 4 Conclusion

In summary, the position dependent spontaneous emission spectra of a \( \Lambda \)-type three-level atom embedded in a defective double-band PBG material have been studied. It was shown that the profile of free space spontaneous emission spectrum of the atom depends on the distance between the atom and the defect location. For the atom far from the defect, we have a three peak spectrum with two dark lines. These dark lines are due to the interaction of the atom with the PBG modes near the edges of the PBG. While for the atom at the defect location, the spontaneous emission spectrum is a two-peak spectrum with a wide dark line, which is the result of coupling between the atom and the defect modes. However, for the atom in the space in between, the spectrum has four peaks with three dark lines due to the coupling of the atom to both PBG reservoir and defect modes. Furthermore, by investigating the average spontaneous emission spectra of the atoms we revealed that the atoms embedded in high (low) dielectric regions far from the defect location effectively couple to the modes of the lower (upper) photonic band. However, the atoms embedded in high dielectric or low dielectric regions near the defect location mainly couple to the defect modes. While, the atoms uniformly embedded in high (low) dielectric regions with a normal distance from the defect location interact with both of defect and lower (upper) photonic band modes.

### References