Back-propagating surface polaritons of a one-dimensional photonic crystal containing single negative metamaterials

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1. Introduction

Recently, special interest was aroused by the outstanding idea of Pendry [1] about the possibility of developing a ‘superlens’, in which the evanescent waves play an important role [2–8]. The amplification of the evanescent waves results from the surface polaritons (SPs), bound to an interface between two different materials. The SPs are those electromagnetic normal modes, which propagate along the interface, and have an evanescent behavior both sides of the interface with their electric and magnetic fields localized near the interface. From this point of view, the SPs are a very sensitive and convenient tool for studying the physical properties of the surfaces (or interfaces). Therefore, the investigations of SPs are indeed important both from a scientific point of view and from a practical one [9]. In recent years, SPs of a semi-infinite isotropic double-negative (DNG) material [10], a DNG slab [11,12] and a multilayer structure of isotropic DNG materials [13] were investigated in detail. Media with both negative permittivity and negative permeability were firstly introduced by Veselago back in the late 1960s [14]. These media are termed left-handed materials (LHMs) or backward wave materials [15], since the electric field (E), the magnetic field (H), and the wave vector (k) of an electromagnetic wave propagating in these media form a left-handed triplet of vectors, instead of a right-handed one observed in conventional right-handed materials (RHM). The physical realization of such LHMs was demonstrated only recently [16] for a novel class of engineered composite materials by using two-dimensional arrays of split-ring resonators and wires [17]. The LHM exhibits many unusual physical properties different from the RHM [11,18–20]. If the permittivity ε and the permeability μ have opposite signs, the medium is called single-negative (SNG) material, which include the epsilon-negative (ENG) medium with ε < 0 (but μ > 0) and the mu-negative (MNG) medium with μ < 0 (but ε > 0) and supports only evanescent wave. All these artificial composites including DNG and SNG materials have exhibited special features in multilayered structures and photonic crystals (PCs) [21–27]. The most well-known feature of PCs is the photonic band gap (PBG), inside which the electromagnetic waves are prohibited to propagate in all directions [28]. However, when appropriately terminated, PCs can support SP modes [29–31], with the frequencies lying inside the PBG.

Most of the work in the area of the linear dispersion characteristics of SPs exciting on the PCs reported in the recent literature has been concerned with the SPs at PCs containing DNG materials [13,31]. However, SNG materials may also possess interesting PBG properties when they are paired in a proper order, namely, ENG-MNG or MNG-ENG layered periodic structures. Therefore, suitably designed PCs based on SNG media may offer exciting possibilities (e.g., omnidirectional band gap [26]) in the design of future photonic devices and components. As a further contribution to the topic of SPs with SNG media, here we present the results of our theoretical analysis of SPs that can be excited at the interfaces between a semi-infinite uniform DNG medium and a 1D-PC containing SNG materials. The benefit of using the periodic multilayer structure consisting of SNG materials is that this structure can provide an omnidirectional photonic gap with constant frequency range independent of the incident angles [26]. Such definite frequency range is interested to...
use simultaneously in LHM and SNG materials. On the other hand, the use of LH metamaterials in the considered structure has the advantage of supporting back-propagating SPs with the negative energy flow.

We show the excitation of special type of transverse structure for TM-polarized SPs with a backward energy flow and determine the existence regions of the SP modes corresponding to light polarization for both ENG-MNG and MNG-ENG periodic structures. The paper is arranged as follows. In Section 2 we explain how to calculate the SP modes exciting on the interface separating semi-infinite DNG medium and a 1D-PC containing SNG materials. The discussion of the dispersion characteristic of the SPs on the plane of the angular frequency versus the propagation constant and existence regions for the back-propagating SPs is illustrated in Section 3. Finally, Section 4 concludes the paper.

2. Model and surface waves dispersion

Referring to Fig. 1, we consider an interface between a semi-infinite DNG medium and a 1D-PC made of two different kinds of SNG materials, and search for the SP modes. For the DNG medium we will employ dispersive forms of the permittivity and the permeability as [26]

\[ \varepsilon_0 = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_0 = 1 - \frac{\omega_m^2}{\omega^2}. \]  

For the ENG layers with thickness \( d_1 \), we use

\[ \varepsilon_1 = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_1 = 3 \]  

and for the MNG layers with thickness \( d_2 \), we consider

\[ \varepsilon_2 = 3, \quad \mu_2 = 1 - \frac{\omega_m^2}{\omega^2}, \]  

where \( \omega \) is the angular frequency of light and \( \omega_p, \omega_m \) are electric and magnetic plasma frequencies, respectively. The DNG and the SNG frequency ranges are determined by the condition \( \omega^2 < \omega_p^2, \omega_m^2 \). To simplify the computational work the electric plasma frequency \( \omega_p \) in the ENG layer and the magnetic plasma frequency \( \omega_m \) in the MNG layer are taken to have the same values \( \omega_p = \omega_m = 10 \text{GHz} \). The values of the magnetic permeability, \( \mu_1 \) in the ENG layer and the electric permeitivity, \( \varepsilon_2 \) in the MNG layer are also taken to be the same. However, these values could be different and do not affect the main features of the results. Smith et al. demonstrated a composite medium, based on a periodic array of split ring resonators (SRRs) and continuous wires, that exhibits a frequency region in the microwave regime (about 5 GHz) with simultaneously negative values of effective permeability and permittivity [16]. It is obvious that individually using of the constituents of the composite medium namely, SRRs and wires medium can provide the MNG and ENG materials, respectively. Hence, it looks reasonable that one can realize a practical multilayer arrangement composed of ENG and MNG layers, so that to operate in given frequency of about 5 GHz.

We consider the propagation of TM-polarized waves with the magnetic field \( H_y = H_y(z) \) in the \( y \) direction (see Fig. 1) which satisfy the following scalar Helmholtz-type equation. Here, \( z \) is the stratification direction. We look for the stationary solutions propagating along the interface with the characteristic dependence \( \sim \exp(-ikx-\beta z/c) \), where \( k \) is the normalized wave number component along the interface, and \( c \) is the speed of light. SP modes are correspond to the localized solutions with the field decaying from the interface in both the directions. In a left-side homogeneous medium \( z < -d_1 \) (see Fig. 1), the fields are decaying provided \( \beta > \omega_0\mu_0 \). In the right-side periodic structure, the waves are the Bloch modes.

\[ E(z) = \Psi(z) \exp(ikz), \]  

where \( k_0 \) is the Bloch wave number, and \( \Psi(z) \) is the Bloch function which is periodic with the period of the photonic structure. In the periodic structure the waves will be decaying provided \( k_0 \) is complex; and this condition defines the spectral gaps of an infinite photonic crystal. For the calculation of the Bloch modes, we use the well-known transfer matrix method [32]. One can find the transfer matrix of TM-polarized waves for a periodic structure containing single negative materials as

\[ M = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \]  

Here

\[ A = e^{ik(d_1 + d_2)} \left( \cosh(k_2d_2) + \frac{1}{2} \left( k_{11} k_{23} - \frac{k_2 k_{12}}{\varepsilon_1 \varepsilon_2} \right) \sinh(k_2d_2) \right), \]  

\[ B = \frac{1}{2} e^{ik(d_1 - 2d_2)} \left( k_{11} k_{22} - \frac{k_2 k_{12}}{\varepsilon_1 \varepsilon_2} \right) \sinh(k_2d_2), \]  

\[ C = -\frac{1}{2} e^{-ik(d_1 - 2d_2)} \left( k_{11} k_{22} - \frac{k_2 k_{12}}{\varepsilon_1 \varepsilon_2} \right) \sinh(k_2d_2), \]  

\[ D = e^{-ikd_1} \left( \cosh(k_2d_2) - \frac{1}{2} \left( k_{11} k_{23} + \frac{k_2 k_{12}}{\varepsilon_1 \varepsilon_2} \right) \sinh(k_2d_2) \right). \]  

Here, \( k = \omega/c, \ v_1, v_2, v_{12i} = k_1 \sqrt{\varepsilon_1 \mu_1 - \beta^2} \) and \( k_{22} = k_1 \sqrt{\varepsilon_2 \mu_2 - \beta^2} \).

We assume that the terminating layer (or a cap layer) of the periodic structure has the width different from the width of other layers of the structure. So that, to clearly separate the periodic bulk from the remaining surface layer, we split the termination layer with width \( d_i \) into two pieces, of lengths \( d_i - d_i + d_i \) (see Fig. 1). Here we study the effect of the width of this termination layer on the SPs and we show that there is a possibility to control the dispersion properties of SPs by adjusting the cap layer thickness \( d_i \) (see Fig. 1). Using boundary conditions of continuity of the tangential components of the electric and magnetic fields at the interface between homogeneous medium and periodic structure [33], we obtain the exact dispersion relation \( k(\beta) \) for
SPs by solving

\[ \frac{q_{B0}}{H_0} = -i \lambda e^{i \theta} B, \]

\[ \frac{k_{z1}}{\mu_1} = -i \lambda e^{i \theta} B, \]

where \( q_{B0} = k_0 \sqrt{\mu_0 \mu_r} \), \( \lambda = (A + D)/2 \pm \sqrt{(A + D)^2/4 - 1} \), and \( B = e^{-2i \omega \mu_0} \). The parameter \( \lambda \) determines the band structure. Regions where \( |\lambda| < 1 \) correspond to real \( k_0 \) and thus to propagating Bloch waves. In regime where \( |\lambda| > 1 \), \( k_0 \) has an imaginary part, therefore the Bloch wave is evanescent, and this regime correspond to forbidden bands (or gaps) of the periodic structure. The band edges are those regimes where \( |\lambda| \approx 1 \).

3. Results and discussion

We analyze the dispersion properties of the SP states in the second PBG of the SNG PC, for two different cases of ENG-MNG and MNG-ENG order of multilayer structures. This PBG is called zero-phase (zero-\( \phi \)) gap resulting from a zero total phase and exists in the periodic structure composed of SNG materials [26]. The zero-\( \phi \) gap condition can be expressed as \( |k_{z1}/d_1| = |k_{z2}/d_2| \). This gap has the properties such as robustness for scaling and omnidirectional reflection [8,9]. The robust property for scaling can be considered as an effect of effective medium theory. Because the frequency realizing SNG is in the microwave region (GHz), and the thickness of the unit cell is generally considered to be around \( 10^{-2} - 10^{-3} \) m, the structure can always satisfy the long wave approximation. As a consequence, the whole periodic structure may be effectively regarded as a slab of material with effective permittivity and permeability. In this structure, when the effective permittivity and effective permeability have different signs at the same frequency, the structure will present an intrinsic forbidden region. Moreover, if the structure undergoes a certain small change of scale and the long wave approximation is still available, the effective permittivity and permeability do not change, thus leads to the robustness for scaling [34]. For a large scaling, the long wave approximation may not be available, and the positions of the band edges and the width of the gap will change. As for the omnidirectional reflection, the zero-\( \phi \) gap will change slightly when the incident angle deviates from zero. However, compared with the usual Bragg gaps which are very sensitive to the incident angle, the zero-\( \phi \) gap is almost independent of the incident angle and is seen to be omnidirectional [26]. These two properties make the SNG materials as useful as the DNG materials. The dispersion curves of the SP modes are shown on the plane of the angular frequency \( \omega \) versus the propagation constant \( \beta \) in Fig. 2. Here, the unshaded regions in Fig. 2 show the second omnidirectional zero-\( \phi \) gap of periodic multilayer structure, while the shaded regions indicate the corresponding pass bands. Each point in dispersion curves indicates the SP mode whose frequency lies inside the zero-\( \phi \) gap and calculated by solving the dispersion relation of Eq. (6) governing to SPs.

Since our studies show that we have only TM-polarized SP modes for the proposed structure (see Fig. 5), so we turn our attention to the TM-polarized SPs and present dispersion characteristic of SP modes for different values of the cap layer thickness \( d_c \) for ENG-MNG and MNG-ENG multilayered structures in Fig. 2(a) and (b), respectively. As one can see from Fig. 2 there are different dispersion curves for different values of cap layer thickness \( d_c \), which describe a possibility to control the dispersion properties of SPs by adjusting the cap layer thickness \( d_c \). Corresponding values of \( d_c \) for dotted, dashed, and solid curves of dispersion are \( d_c = 0.2 d_1, 0.8 d_1 \), and \( 2 d_1 \), respectively. Fig. 2 shows that there are a special SP mode at \( \beta = 1.5 \) and \( \omega = 5.0 \text{GHz} \) which always can occur for any given cap layer thickness in both ENG-MNG and MNG-ENG structures. Such a mode is due to the condition \( B = 0 \) which is satisfied at the frequency \( \omega = 5.0 \text{GHz} \) for the chosen parameters.

The existence of the localized SPs in the used structure can be explained using perturbation theory [35]. According to this theory, the periodic structure with alternating layers can be considered as a system of interacting waveguides. These waveguides are identical to each other except for the one near the surface of the PC. The interaction strength between the waveguides depends on the separation between the neighboring waveguides. When the separation is infinite, there is no interaction, and the waveguides can be considered as independent of each other. In this case, for a given \( \beta \), the eigenvalues of \( k \) fall into two groups: one is an infinitely degenerate state; the other is a nondegenerate state which is corresponds to the guide near the surface. As the waveguides are brought together, the interaction between the waveguides causes the eigenvalues to split. As the eigenvalues split, the propagation band for the infinite structure is
fully occupied by the states originating in the infinitely degenerate state. As a result, the nondegenerate state corresponding to the guide near the surface will be pushed out of the propagation band. The only place where this state can be accommodated is in the band gap. These waves are localized near the surface because their corresponding wave numbers are in the forbidden band gap.

To show the localized nature of the SP modes, the mode profiles of points (1) and (2) of Fig. 2 are shown in Fig. 3, where we plotted the profiles of the mode (1) from ENG-MNG structure, and the mode (2) from MNG-ENG structure.

As stated in Fig. 3, the localized modes have evanescent nature and due to this evanescent nature, they will not interact directly with an incoming plane wave. So, they can be excited by the ATR method. This technique has previously been invoked for the investigation of various types of surface polaritons, e.g., plasmon-polaritons in metals [36,37], phonon-polaritons in ionic crystals [38,39], exciton-polaritons in semiconductors [40,41] and magnon-polaritons in magnetic materials [42,43]. However, for the sake of brevity we do not consider this method here and refer the reader to the relevant references [10,11].

The energy flow of SPs for considered ENG-MNG and MNG-ENG structures has same behavior with negative values. To demonstrate this, in Fig. 4 we plot the total energy flow in the modes as a function of the wave number \( \beta \). We see from Fig. 4 that the total energy flow of all SP modes is backward with respect to the wave vector. Therefore, the modes of both ENG-MNG and MNG-ENG structures (with different values of cap layer thicknesses \( d_c \)) are back-propagating SP modes.

Finally, in Fig. 5 we fix the thicknesses of the MNG layer and study the dependence of the zero-\( \phi \) PBG and SP modes on the ratio of the thicknesses of the SNG layer to MNG layer for (a) ENG-MNG (b) MNG-ENG structures. Here, \( d_1 = d_{\text{ENG}}, d_2 = d_{\text{MNG}} \) for ENG-MNG structure and \( d_1 = d_{\text{MNG}}, d_2 = d_{\text{ENG}} \) for MNG-ENG structure. The dotted, dashed, and solid lines correspond to the SP modes with \( d_c = 0.2 d_1, 0.8 d_1, \) and \( 1.5 d_1 \) cm, respectively. Here, \( \beta = 1.86 \), and the other parameters are the same as Fig. 1. The unshaded regions are the omnidirectional zero-\( \phi \) PBG in which the existence of TM-polarized or TE-polarized SP modes is indicated. Fig. 5 shows that in the cases of ENG-MNG and MNG-ENG structures the low and high edges of gap are affected by the ratio of the thicknesses, and we have only

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**Fig. 3.** (Color online) Examples of the backward TM-polarized SP modes. (a) ENG-MNG structure: \( \omega = 4.919 \text{GHz}, \beta = 1.72, \) and \( d_c = 0.8 d_1 \). (b) MNG-ENG structure: \( \omega = 4.562 \text{GHz}, \beta = 1.94, \) and \( d_c = 2 d_1 \). Modes (a) and (b) correspond to the points (1) and (2) in Fig. 2, respectively. The other parameters are the same as the Fig.1.

**Fig. 4.** (Color online) Total energy flow of SP modes versus \( \beta \) in the (a) ENG-MNG and (b) MNG-ENG periodic structures for different \( d_c \). Dotted, dashed, and solid curves show the energy flow of the SP modes for \( d_c = 0.2 d_1, 0.8 d_1, \) and \( 2 d_1 \), respectively.
TM-polarized SP modes for large ENG layers to values more than the thicknesses of MNG layers. There ENG-MNG and MNG-ENG structures by increasing the thicknesses of structures. The dotted, dashed, and solid lines correspond to the SP modes with Figs. 5(a) and (b), it is possible to find TE-polarized SP modes (with thickness of MNG layer, for example the case of structure that we TM-polarized SP modes when the thickness of SNG layer less than (Color online) The dependence of the zero-

Fig. 5. (Color online) The dependence of the zero- \( \omega \) PBG and SP modes on the ratio of the thicknesses of the SNG layer to MNG layer for (a) ENG-MNG (b) MNG-ENG structures. The dotted, dashed, and solid lines correspond to the SP modes with \( d_1=0.2d_1, 0.8d_1, \) and 1.5 \( d_1 \) cm, respectively. Here, \( \beta =1.86, \) and the other parameters are the same as Fig. 1.

TM-polarized SP modes when the thickness of SNG layer less than thickness of MNG layer, for example the case of structure that we considered in this paper. However, as one can clearly see from Figs. 5(a) and (b), it is possible to find TE-polarized SP modes (with the electric field \( \textbf{E}=\textbf{E}(y) \) in the y direction, see Fig. 1) for the both ENG-MNG and MNG-ENG structures by increasing the thicknesses of ENG layers to values more than the thicknesses of MNG layers. There are no limitation in the case of ENG-MNG structure on the existence region of TM-polarized SP modes for large \( d_1, \) whilst in the case of MNG-ENG structure the existence regions of TM-polarized SP modes decrease by increasing of thickness of cap layer \( d_1, \) (see solid line in Fig. 5(b) for TM-polarized modes). Our studies show that the existence regions of the SP modes depend on the ratio of the thicknesses of two ENG and MNG layers. These properties of SP modes can be explained from the effective medium approximation parameters are the same as Fig. 1.

4. Conclusion

We have presented a theoretical study of TM-polarized electromagnetic SPs supported by an interface between a DNG metamaterial and a 1D-PC containing alternative ENG-MNG or MNG-ENG layers. We have demonstrated that in the presence of a DNG material there are only back-propagating TM-polarized (or TE-polarized) kind of the SPs depending on the ratio of the thicknesses of the two periodic stacking layers. When the ratio of thicknesses of the SNG layer to MNG layer is less than one (or more than one), we face with only TM-polarized (TE-polarized) SP modes for both structures. We believe that our results can be useful in the design of future photonic devices and components based on the properties of SPs.

References