Localized modes in defective multilayer structures

S. Roshan Entezar1 and A. Namdar1,2
1Physics Faculty, University of Tabriz, P.O. Box 5166614766, Tabriz, Iran
2Department of Physics, Azarbaijan University of Tabriz Moallem, P.O. Box 5167618949, Tabriz, Iran

(Received 25 January 2009; revised manuscript received 22 April 2009; published 17 July 2009)

In this paper, we investigate the localized surface modes in a defective multilayer structure. We show that the defective multilayer structure can support two different kinds of localized modes, depending on the position and the thickness of the defect layer. In one kind, the modes are localized at the interface between the multilayer structure and a homogeneous medium (the so-called surface modes). While, in the other one, the modes are localized at the defect layer (defect-localized modes). We reveal that in the presence of the defect layer, the dispersion curve of the surface modes is pushed to the lower (upper) edge of the photonic band gap when the homogeneous medium is a left-handed (right-handed) material. Therefore, the existence regions of the surface modes are restricted due to the defect layer. Moreover, the effect of defect on the energy flow velocity of the surface modes is discussed.

DOI: 10.1103/PhysRevA.80.013814 PACS number: 42.70.Qs, 41.20.Jb

I. INTRODUCTION

The advent of photonic crystal (PC) materials [1,2] excited a lot of interest toward the existence of surface modes (SMs) at the interfaces of such materials [3–6]. These modes remain localized around the PC surface in a manner similar to surface plasmons on metal slabs [7]. In real materials, PCs are always finite in size and, therefore, SMs can exist. The existence of SMs can directly affect the performance and the efficiency of PCs in applications. Thus, it is important to study the SMs in PCs. The first experimental observation of such SMs came by Robertson et al. [3]. In the experiment, the authors employed a standard attenuated total reflection (ATR) setup, widely used for surface-plasmon observations on metals [10,11]. The majority of the subsequent theoretical studies focused on the existence of such SMs in various PC structures. It was found that the frequency and in many cases even the mere existence of SMs are strongly influenced by the way the periodic PC is terminated [4,5]. Nevertheless, the initial acute interest for SMs had somewhat subsided until recently. The need to understand and to engineer SMs came back to light when these deemed to play a key role in newly discovered PC phenomena. In particular, it was found that SMs can strongly influence the subdiffraction focusing properties of PC-based slab superlenses [12–15]. Moreover, coupling to such SMs in PC subwavelength-width waveguides leads to a highly directional exit beam [16,17]. Despite the intensive research on PC surface phenomena [3–6], one aspect of the SM propagation remains unexplored. When the periodicity of PC is broken by introducing a defect into a PC, a defect mode will appear inside the photonic band gap (PBG) due to change in the interference behavior of light, whose properties would be determined by the nature of the defect. The introduction of defect layers in one-dimensional PCs can create defect modes within the PBGs, just as defect layers in semiconductor superlattices may result in electron defect states in the band gaps. A natural question arises: what is the effect of defect on the SMs?

In this paper, we investigate the effect of a defect layer on the SMs at the interface between a multilayer structure and a homogeneous medium (the surface of the structure). We show that two different kinds of localized modes can be created in the defective multilayer structure depending on the position and the thickness of the defect layer. One kind has a peak intensity at the surface of the structure (SM). But, the other one is localized at the defect layer which we call it the defect-localized mode (DLM). Furthermore, it is shown that the existence region of the SMs and the DLMs depends on the position and the thicknesses of the defect layer. As well, we show that the energy flow velocity of the SMs has not been considerably affected due to the existence of the defect layer in the periodic multilayer structure. However, the energy flow velocity of the DLMs is affected by the position and the thickness of the defect layer. In Sec. II, we introduce the model of the system under consideration. In Sec. III, the properties of localized modes are studied. Finally, Sec. IV concludes with brief comments.

II. FORMALISM

We wish to describe the localized modes that form at the interface between a homogeneous medium of low refractive index, $n_0=\sqrt{\varepsilon_0}\mu_0$, and a defective multilayer structure with layers of refractive indices $n_1=\sqrt{\varepsilon_1}\mu_1$, $n_2=\sqrt{\varepsilon_2}\mu_2$, thicknesses $d_1,d_2$, and period $d=d_1+d_2$. Here we assume the homogeneous medium is a left-handed medium and we adopt the typical values $\varepsilon_0=-1$, $\mu_0=-1$, $\varepsilon_1=2.25$, $\mu_1=1$, $\varepsilon_2=4$, $\mu_2=1$, $\bar{d}_1=\frac{d_1}{\bar{d}}=0.63$, and $\bar{d}_2=\frac{d_2}{\bar{d}}=0.37$. We suppose that the defect layer with dimensionless thickness $\bar{d}_d=\frac{d_d}{\bar{d}}$ and refractive index $n_d=\sqrt{\varepsilon_d}\mu_d$ is located after $\eta$ ($\eta=0,1,2,\ldots$) complete periods of multilayer structure; hence, $\eta$ describes the position of the defect layer in the structure. The order of magnitude of $d$ depends on the desired spectral range. For the microwave region with frequency around 5–12 GHz (the suitable range for the left-handed metamaterials (LHM) [18]), $d$ is on the order of cm. While, $d$ can be on the order of mm-nm for the optical range (for example, see the experimental work in Ref. [19]). On the other hand, because of the recent developments in designing negative refractive index
material at the optical range [20–23], the use of left-handed metamaterial as a homogeneous medium is realizable. In what follows, we use the dimensionless wave vectors. We choose a coordinate system in which the layers have normal vector along OZ (the z axis of the coordinate system, see Fig. 2) and we consider the propagation of monochromatic TE-polarized waves described by [24]

\[
E = E_y(z)\hat{e}_y e^{i(k_{Bz}\omega - \omega t)},
\]
\[
H = [H_z(z)\hat{e}_z + i H_x(z)\hat{e}_x] e^{i(k_{Bz}\omega - \omega t)},
\]
with the electric field E in the y direction. Here, \( \omega \) is the angular frequency; \( k = (\omega/\gamma)d \) is the dimensionless vacuum wave number; \( \beta = k_x \); \( k_x \) is the x component of the dimensionless wave vector of modulus \( kn \); and \( x, z \) are dimensionless coordinates. We look for stationary solutions propagating along the interface which satisfy the following scalar Helmholtz-type equation:

\[
\frac{d^2}{dz^2} - k^2 \frac{\omega^2}{c^2} e(z) \mu(z) - \frac{1}{\mu(z)} \frac{d\mu}{dz} \frac{dE_y}{dz} = 0.
\]

In the periodic structure, the waves are the Bloch modes \( E_y(z) = \psi(z) e^{ikz}, \) where \( K_0 \) is the dimensionless Bloch wave number, and \( \psi(z) \) is the Bloch function, which is periodic with the period of the photonic structure (see Ref. [25]). In the periodic structure the waves will be decaying provided \( K_0 \) is complex, and this condition defines the spectral band gaps of an infinite multilayer structure. For the calculation of the Bloch modes, we use the well-known transfer-matrix method [25]. To find the localized modes, we take the solutions of Eq. (2) in a homogeneous medium (\( z \geq 0 \)) as

\[
E_y(z) = b e^{i(q_0z)}
\]

and in the \( p \)th cell of the periodic structure before the defect layer (\( p = 0, 1, \ldots, \eta - 1 \)) as [26]

\[
E_y(z) = (a_{p+1} e^{i(k_1(z-p))} + b_{p+1} e^{-i(k_1(z-p))}),
\]
when \( p \leq z \leq p + \bar{d}_1 \), and

\[
E_y(z) = (c_{p+1} e^{ik_2(z-p)} + d_{p+1} e^{-ik_2(z-p)}),
\]
when \( p + \bar{d}_1 \leq z \leq (p + 1) \). Similarly, in the \( q \)th cell of the periodic structure after the defect layer (\( q = \eta, \eta + 1, \ldots \)), we use

\[
E_y(z) = (a_{q+1} e^{i(k_1(z-q))} + b_{q+1} e^{-i(k_1(z-q))}),
\]
when \( q + \bar{d}_d \leq z \leq q + \bar{d}_d + \bar{d}_1 \), and

\[
E_y(z) = (c_{q+1} e^{ik_2(z-q)} + d_{q+1} e^{-ik_2(z-q)}),
\]
when \( q + \bar{d}_d + \bar{d}_1 \leq z \leq (q + 1) + \bar{d}_d \). For the defect layer (\( \eta \leq z \leq \eta + \bar{d}_d \)), we use

\[
E_y(z) = (a_\eta e^{ik_2(z-\eta)} + b_\eta e^{-ik_2(z-\eta)}).
\]

Here, \( q_0 = k/\beta^2 - n_0 \), \( k_1 = k/\sqrt{n_1^2 - \beta^2} \), and \( k_2 = k/\sqrt{n_2^2 - \beta^2} \). Then we satisfy the conditions of continuity of the tangential components of the electric and the magnetic fields at the interfaces of the structure as

\[
b = a_1 + b_1,
\]
when \( z = 0 \), and

\[
c_\eta e^{ik_2} + d_\eta e^{-ik_2} = a_d + b_d,
\]
when \( z = \eta \), and

\[
a_\eta e^{ik_2} + d_\eta e^{-ik_2} = a_{\eta + 1} + d_{\eta + 1},
\]
when \( z = \eta + \bar{d}_d \). By solving these equations and using the properties of the transfer matrix, we can obtain the exact dispersion relation \( k = k(\beta) \) for TE-polarized localized modes by numerically solving the following dispersion condition:

\[
q_0 \mu_1 = -i \frac{1}{(A + B)U_{\eta - 1} - U_{\eta - 2})(\lambda - A) - (A + B)^* U_{\eta - 1} - U_{\eta - 2}}{\beta},
\]
where \( A, B \) are the elements of the transfer matrix of the multilayer structure; \( \lambda \) is the eigenvalue of the transfer matrix \( \begin{bmatrix} A & B \\ B^* & A \end{bmatrix} \) [24]; \( \beta = \text{Re} - 2ik_2, \) and \( U_{\eta} = \sin[(\eta + 1)K_0]/\sin(K_0) \).

**III. NUMERICAL RESULTS AND DISCUSSIONS**

In the following, we want to discuss the effect of the defect layer on the dispersion properties of the localized modes. In the used structure, we assumed that the homogeneous medium is a LHM with \( n_0 = -1 \). For comparison, we also investigated the dispersion of the corresponding localized modes in the structure when the homogeneous medium is replaced with the vacuum (\( n_0 = 1 \)). To do this, we plotted the first two sides of the dispersion condition [Eq. (15)] vs \( k \) in the first spectral band gap in Fig. 1. In this figure the solid and the dotted lines show the left-hand side of Eq. (15) for the cases \( n_0 = -1 \) and \( n_0 = 1 \), respectively, while the dashed lines

\[
E_y(z) = (c_{p+1} e^{i(k_2(z-p))} + d_{p+1} e^{-i(k_2(z-p))}),
\]
represent the right-hand side of Eq. (15). Here we take the following values: $d_0=0.3$, $\eta=3$, and $\beta=1.14$. Unlike the defectless structure [27], we see that the dispersion condition has two solutions (points 1 and 2 in Fig. 1). So, there are two different modes for a given $\beta$. To describe these modes, in Fig. 2 we plotted the intensity distributions and the transverse structures of these modes as functions of $z$. As one can see, both of these modes are localized modes. But, mode 1 is localized at the surface of structure (SM) [see Fig. 2(a)], while mode 2 is localized at the defect layer (DLM) [see Fig. 2(b)].

The existence of the localized waves in the defective structure can be explained using perturbation theory [28]. According to this theory, the periodic structure with alternating layers of different refractive indices can be considered as a system of interacting waveguides. These waveguides are identical to each other except for those near the surface of PC and the defect layer. The interaction strength between the waveguides depends on the separation between the neighboring waveguides. When the separation is infinite, there is no interaction, and the waveguides can be considered as independent of each other. In this case, for a given $\beta$, the eigenvalues split, the propagation band for the infinite layer. As the waveguides are brought together, the interaction between the waveguides causes the eigenvalues to split. As the eigenvalues split, the propagation band for the infinite structure is fully occupied by the states originating in the infinitely degenerate state. As a result, the nondegenerate states corresponding to the guides near the surface and the defect layer will be pushed out of the propagation band. The only place where these states can be accommodated is in the band gap. These waves are localized near the surface and the defect layer because their corresponding wave numbers are in the forbidden band gap.

Our investigations indicate that the existence regions of the SMs and the DLMs strongly depend on the position of defect layer. To show this, in Fig. 3 we plotted the dispersion curves of the localized modes in the first spectral gap as a function of $\beta$ for different defect positions. For a defect layer near the surface ($\eta=1,2$), the coupling between the guide near the surface and the guide near the defect is strong. As a result of this strong coupling, the eigenvalues of $k$ corresponding to the guide near the surface (SMs) are pushed to the edge of the band gap. Accordingly, we can have only the DLMs [see Figs. 3(a) and 3(b)]. By increasing $\eta$, the coupling between the guide near the surface and the guide near the defect becomes weaker, so that the two eigenvalues of $k$ corresponding to the SMs and the DLMs appear inside the band gap [see Fig. 3(c)]. For the case of sufficiently large $\eta$, the structure acts as a defectless structure due to enough periods before the defect layer. So, the DLM disappears in the benefit of the appearance of the SM [see Fig. 3(d)]. In Fig. 3 we also studied the dispersion property of two different homogeneous media (i.e., LHM and vacuum). We see that when the homogeneous medium is a LHM ($n_0=-1$), the dispersion curve of the SMs settle near the lower edge of the photonic band gap and finally disappears by decreasing the distance of the defect layer from the surface of the structure.
Fig. 4. (Color online) The reflectivity of a defectless multilayer structures with (a) $\beta=1.09$ and (b) $\beta=1.29$ and a defective multilayer structures with (c) $\beta=1.09$ and (d) $\beta=1.29$ as functions of dimensionless vacuum wave number $k$. Here, the thin lines show the case for $n_0=-1$ and the thick lines show the case for $n_0=1$. The other parameters are $\bar{d}_d=0.3$, $\eta=3$, and $\bar{n}=3$.

This is in contrast to the case where the LHM is replaced with the vacuum. For this case the dispersion curve of the SMs moves toward the upper edge of the photonic band gap.

As stated in Fig. 2, the localized modes have evanescent nature and due to this evanescent nature, they will not interact directly with an incoming plane wave. So, they can be excited by the ATR method. This technique has previously been invoked for the investigation of various types of surface polaritons, e.g., plasmon polaritons in metals [10,11], phonon polaritons in ionic crystals [29,30], exciton polaritons in semiconductors [31,32], and magnon polaritons in magnetic materials [33,34]. We consider the ATR geometry shown in Fig. 1 of Ref. [35]. Here, the uniform medium represents a gap layer of width $L$ that separates dielectric and layered structure. For an incident angle larger than the angle of the total internal reflection, the electromagnetic field incident from an optically dense medium (dielectric) with refractive index $\bar{n}>n_0$ will penetrate the gap as an evanescent wave, which can interact with the evanescent localized modes. We have calculated the reflectivity of the ATR geometry, using classical electromagnetic theory. A calculated ATR spectrum for the cases of defectless and defective multilayer structures is shown in Fig. 4 for two different incident angles [or two different $\beta=\bar{n} \sin(\theta)$]. Here, the shaded regions show the first spectral band gap of the multilayer structure. In Figs. 4(a) and 4(b) there is only a deep in the ATR spectrum in the spectral band gap corresponding to a SM. However, in Fig. 4(c) we reveal two sharp deeps in the ATR spectrum corresponding to a DLM and a SM. In Fig. 4(d), we can see only a deep in the spectral band gap related to a DLM.

Our investigations show that besides the position of the defect layer ($\eta$), its thickness ($\bar{d}_d$) can affect the existence region for the SMs and the DLMs. To show this, we plotted the reflectivity of the ATR geometry as a function of $k$ for different thicknesses of defect layer with a fix position in Fig. 5. It is seen that for a thin defect layer, the multilayer structure behaves like as a defectless structure (see solid lines in Fig. 5) [27]. Increasing the thickness of the defect layer $\bar{d}_d$ will push the SM to the lower (upper) edge of the band gap in the case of $n_0=-1$ ($n_0=1$). Finally, the SM will disappear for sufficiently thick defect layer, and the DLM will appear within the band gap. In other words, for a large $\bar{d}_d$ the structure loses the ability to support the SMs. Therefore, the modes localize at the defect layer. To summarize the effect of thickness of defect layer on the existence region for the SMs and the DLMs, in Fig. 6 we plotted the dispersion curves of the SMs and the DLMs for a given $\beta$ and different $\eta$ as a function of thickness of defect layer $\bar{d}_d$. As one can see from Fig. 6, the SMs are pushed more and more to the edge of the band gap by increasing the thickness of defect layer. Nevertheless, the effect of the defect layer on the SMs is reduced by increasing the distance between the defect layer and the surface of the structure.

Finally, it is of interest to know the effect of defect layer on the energy flow velocity of the localized modes. It is well known that the energy flow velocity $v_E$ relates the total energy flow $\langle S \rangle$ to the total energy density as $v_E=\frac{\langle S \rangle}{\langle E \rangle}$. The
Here, the localized modes in defective multilayer structures in Figs. 7 and 8, respectively. It is seen that in the presence of uniform LHM, the energy flow velocity of the DLMs depends on the position and the thickness of the defect layer [see Figs. 7(b) and 8(b)].

**IV. CONCLUSION**

Briefly, we have investigated the dispersion characteristics of the localized modes in a defective multilayer structure. In particular, we have explained that the structure can support two different kinds of localized modes which are localized at the surface of the structure (SM) and the position of the defect (DLM), respectively. Moreover, we have revealed that the existence regions of the SMs and the DLMs depend on the position and the thickness of the defect layer. It was shown that the dispersion curves of the SMs settle near the lower (upper) edge of the photonic band gap when the uniform medium is a LHM (vacuum) and finally disappear by decreasing the distance of the defect layer from the surface of the structure. As well, it was shown that the existence of the defect in the structure has no considerable effect on the energy flow velocity of the SMs.

**ACKNOWLEDGMENT**

The authors thank Professor Yuri S. Kivshar for valuable suggestions and useful guidance.


