LOCALIZED WAVES AT THE SURFACE OF A SINGLE-NEGATIVE PERIODIC MULTILAYER STRUCTURE

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Abstract—The localized surface waves at the interface between an uniform right-handed material and a semi-infinite periodic multilayer structure containing alternate \( \varepsilon \)-negative (ENG) and the \( \mu \)-negative (MNG) layers have been investigated. We demonstrated that this system can only support TE or TM-polarized surface waves depending on the relative thicknesses of the ENG and the MNG layers. These localized modes can have two different transverse structure related to the order of the ENG and the MNG layers; one with a hump at the interface between uniform material and the cap layer and the other one with a hump at the interface between the cap layer and the periodic multilayer structure.
1. INTRODUCTION

Recently, metamaterials as specially engineered media with unconventional response functions, has attracted a great deal of attention. Particularly, media in which both of the material parameters, permittivity ($\varepsilon$) and permeability ($\mu$), can attain negative real parts in a certain frequency band, have been studied by numerous groups (see e.g., [1]). When both material parameters possess negative real parts, such double-negative (DNG) media can support wave propagation and exhibit the unusual phenomenon of negative refraction. Besides the DNG materials, we can also have materials in which only one of the two material parameters $\varepsilon$ and $\mu$ is negative [2]. These so-called single-negative (SNG) materials support evanescent wave in order to maintain positive definite energy density. Because $\varepsilon$ and $\mu$ are frequency dependent, only within a certain frequency range we have $\varepsilon < 0$ and $\mu > 0$ (epsilon-negative) or $\varepsilon > 0$ and $\mu < 0$ (mu-negative), which is called the SNG frequency range. The DNG and SNG metamaterials, formed by embedding arrays of metallic split-ring resonators and wires in a host medium [2], have been successfully constructed in the microwave regime by several groups, and some of their unusual properties (e.g., negative refraction) have been experimentally demonstrated [1]. All these artificial composites (including DNG and SNG materials) have exhibited special features in photonic crystals (PCs) [3–5]. The essential property of PCs is the photonic band gap (PBG) structure originated from the consequence of Bragg scattering. Such a Bragg gap in conventional PCs is strongly dependent on the lattice constant, and the incident angle and polarization of light, and is affected by disorder of devices. Jiang et al. [5] found that the ENG-MNG multilayered periodic structures can possess a new type of photonic band gap (called the effective zero-phase gap), which is distinct from the Bragg gap but similar to the zero average index gap [6–8]. Such a zero-phase gap is an omnidirectional band gap and is insensitive to the incident angles and polarizations of the incident light [9–11].

The excitation of Surface waves (SWs) have recently been proposed [12–14] as a way to efficiently inject light into a PC waveguide, or to extract a focussed beam from a channel. SWs on a 1D-PC were observed almost 30 years ago [15, 16]. The basic theory was developed at that time by Yariv and Yeh [17, 18]. The effect of varying the thickness of the termination layer has been measured experimentally [19] and a sensor based on the properties of SWs has been proposed and demonstrated [20]. In parallel, numerical calculations for SWs in the band gaps have been performed, [21, 22]. In this paper, we study the properties of linear SWs at the interface
between uniform right-handed (RH) materials and a semi-infinite 1D-PCs containing alternate ENG and MNG (or MNG and ENG) layers, and demonstrate a number of unique features of SWs in the SNG band gap. It is found out that in the considered periodic structure with ENG-MNG (or MNG-ENG) arrangement we can only excite the TE-polarized surface modes (SMs). These SMs can have two different transverse structure at the second SNG band gap of the photonic crystal depending on the type of arrangement of the layers of PC (ENG-MNG or MNG-ENG). In the ENG-MNG arrangement, the modes have a hump at the interface between the cap layer and the photonic crystal, while in the MNG-ENG arrangement the modes have a hump at the interface between uniform material and the cap layer. We show that in the ENG-MNG arrangement, the dispersion curves can be nearly omnidirectional for thick cap layers. In Sec. 2, we introduce the model of the system under consideration. In Sec. 3, the properties of SWs are studied. Finally, Sec. 4 concludes with brief comments.

2. THEORETICAL MODEL

In this section, we wish to describe SMs that form at the interface between a RH medium of low refractive index, \( n_0 = \sqrt{\varepsilon_0 \mu_0} \), and a semi-infinite 1D-PCs containing SNG materials. We assume that each cell of PCs consists of the ENG-MNG or the MNG-ENG layers with the thickness \( d_i \), relative permittivity \( \varepsilon_i \) and permeability \( \mu_i \) (\( i = 1, 2 \)) (see Fig. 1). Now, we suppose that relative permittivity and permeability in the ENG materials are given by [3]

\[
\varepsilon_i = 1 - \frac{\omega_{ep}^2}{\omega^2 + i\omega \Gamma_e}, \quad \mu_i = a,
\]

and those in the MNG materials are given by

\[
\varepsilon_i = b, \quad \mu_i = 1 - \frac{\omega_{mp}^2}{\omega^2 + i\omega \Gamma_m},
\]

where \( \omega \) is the angular frequency, \( \omega_{ep} \) and \( \omega_{mp} \) are the electronic plasma frequency and the magnetic plasma frequency, respectively. Both \( \Gamma_e \) and \( \Gamma_m \) are the dissipation factors. The frequency \( \omega \) in the above equations is measured in \( 10^9 \text{rad/s} \). Here both positive constants \( a \) and \( b \) are assumed to be \( a = b = 3.0 \). Both \( \omega_{ep} \) and \( \omega_{mp} \) are set to be \( 10 \times 10^9 \text{rad/s} \) [5]. For simplicity, we neglect the damping in the permittivity and permeability (i.e., \( \Gamma_e = \Gamma_m = 0 \)), as adopted in previous works [5, 9, 23]. It should be mentioned that the parameters
\( \varepsilon_i \) in Eq. (1) and \( \mu_i \) in Eq. (2) diverge under the limit of \( \omega \to 0 \), so it has no physical meanings in the low-frequency limit. In the frequency range of \( \omega < \omega_{\text{ep}}, \omega_{\text{mp}} \), either \( \varepsilon \) or \( \mu \) is negative in each layer, that is, to say we have SNG materials. So, in the SNG frequency range of \( \omega \), we have \( \varepsilon_1 < 0, \mu_1 > 0, \) and \( \varepsilon_2 > 0 \) and \( \mu_2 < 0 \) for the periodic structure consists of the ENG-MNG layers. While, for a periodic structure consists of the MNG-ENG layers \( \varepsilon_1 > 0, \mu_1 < 0, \) and \( \varepsilon_2 < 0 \) and \( \mu_2 > 0 \). Hence, in each layer the refractive index is a pure imaginary number and the electromagnetic fields are evanescent. As shown in Fig. 1, the crystal is capped by a layer of the same material but different width, \( d_c \) as an adjusting parameter. We consider the propagation of TE-polarized waves described by [24]

\[
E = E_y(z)\hat{e}_y e^{i(k\beta_x - \omega t)}, \quad (3)
\]

\[
H = (H_x(z)\hat{e}_x + H_z(z)\hat{e}_z)e^{i(k\beta_x - \omega t)},
\]

with the electric field \( E \) in the \( y \) direction (the dielectric layers are in the \( x-y \) plane and the \( z \) direction is normal to the interface of each layer) (see Fig. 1). Here \( k = \omega/c \) is the vacuum wavenumber. For a TE-polarized wave entering the periodic multilayer structure from a uniform medium, the propagation constant \( \beta = k_{\parallel} \) is conserved throughout all interfaces (where \( k_{\parallel} \) is the tangential component of the wave vector). We look for stationary solutions propagating along the

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**Figure 1.** (Color online) Geometry of the problem. a) The ENG-MNG arrangement with the cap layer of the ENG material and \( d_1 = d_{\text{ENG}}, d_2 = d_{\text{MNG}}, \varepsilon_1 = \varepsilon_{\text{ENG}}, \mu_1 = \mu_{\text{ENG}}, \varepsilon_2 = \varepsilon_{\text{MNG}}, \mu_2 = \mu_{\text{MNG}} \); b) The MNG-ENG arrangement with the cap layer of the MNG material and \( d_1 = d_{\text{MNG}}, d_2 = d_{\text{ENG}}, \varepsilon_1 = \varepsilon_{\text{MNG}}, \mu_1 = \mu_{\text{MNG}}, \varepsilon_2 = \varepsilon_{\text{ENG}}, \mu_2 = \mu_{\text{ENG}} \). Here \( \varepsilon_0 = 1, \mu_0 = 1 \).
interface which satisfy the following scalar Helmholtz-type equation

\[
\left[ \frac{d^2}{dz^2} - k_z^2 + \frac{\omega^2}{c^2}\varepsilon(z)\mu(z) - \frac{1}{\mu(z)} \frac{d\mu}{dz} \frac{d}{dz} \right] E_y = 0. \tag{4}
\]

In the right side periodic structure, the waves are the Bloch modes

\[ E_y(z) = \psi(z) e^{iK_b z} \]

where \(K_b\) is the Bloch wavenumber, and \(\psi(z)\) is the Bloch function, which is periodic with the period of the photonic structure (see [18]). In the periodic structure the waves will be decaying provided \(K_b\) is complex; and this condition defines the spectral band gaps of an infinite photonic crystal. For the calculation of the Bloch modes, we use the well-known transfer matrix method [18]. To find the SMs, we take solutions of Eq. (4) in a homogeneous medium and the Bloch modes in the periodic structure and satisfy the conditions of continuity of the tangential components of the electric and magnetic fields at the interface between the homogeneous medium and periodic structure [25]. In this way, we can obtain the exact dispersion relation \(\omega = \omega(\beta)\) for TE-polarized SWs by numerically solving the following dispersion condition for the SMs:

\[
\frac{q_0/\mu_0}{k_1/\mu_1} = -i \frac{\lambda - A - \tilde{B}}{\lambda - A + \tilde{B}}. \tag{5}
\]

Here

\[
\lambda = \frac{A + D}{2} \pm \sqrt{\left( \frac{A + D}{2} \right)^2 - 1},
\]

\[
A = e^{k_1d_1} \left( \cosh(k_2d_2) + \frac{1}{2} \left( x + \frac{1}{x} \right) \sinh(k_2d_2) \right),
\]

\[
\tilde{B} = \frac{1}{2} e^{k_1(d_1 - 2d_2)} \left( x - \frac{1}{x} \right) \sinh(k_2d_2),
\]

\[
D = e^{-k_1d_1} \left( \cosh(k_2d_2) - \frac{1}{2} \left( x + \frac{1}{x} \right) \sinh(k_2d_2) \right),
\]

where \(k_1 = k \sqrt{\beta^2 - \varepsilon_1\mu_1}, \ k_2 = k \sqrt{\beta^2 - \varepsilon_2\mu_2}, \ q_0 = k \sqrt{\beta^2 - n_0^2}\) and \(x = \frac{k_2/\mu_2}{k_1/\mu_1}\). These equations are valid only for the specific condition of \(\varepsilon_1 < 1, \mu_1 > 1, \varepsilon_2 > 1, \mu_2 < 1\) or \(\varepsilon_1 > 1, \mu_1 < 1, \varepsilon_2 < 1, \mu_2 > 1\).

3. RESULTS AND DISCUSSION

In the following, we summarize the dispersion properties of the SMs in the second SNG band gaps which is in the microwave range suitable for the SNG metamaterials.
Figure 2. (Color online) Dispersion property of the TE-polarized SWs at the second SNG band gap for different thicknesses of the cap layer \(d_c\): (a) \(d_c = 0.01d_1\) (dotted lines), \(d_c = 1d_1\) (dashed lines) and \(d_c = 3d_1\) (solid lines) in a) the ENG-MNG arrangement and b) the MNG-ENG arrangement. Here the unshaded regions show the first and the second band gaps of the periodic multilayer structure with SNG constituents, while the shaded regions indicate the corresponding pass bands. In the used structure \(d_{ENG} = 0.6\text{ cm}, \ d_{MNG} = 0.8\text{ cm}\) and \(\frac{d_{ENG}}{d_{MNG}} < 1\).

In Fig. 2, we plotted the dispersion curves of TE-polarized SMs on the plane of \(\omega\) versus the propagation constant \(\beta\) for different thickness of the cap layer \(d_c\). Here we used the structure with \(d_{ENG} = 0.6\text{ cm}\) and \(d_{MNG} = 0.8\text{ cm}\) (i.e., \(\frac{d_{ENG}}{d_{MNG}} < 1\)). It is seen that the dispersion curve of TE-polarized SMs for a given \(d_c\) depends on the type of arrangement of SNG layers: ENG-MNG arrangement (Fig. 2(a)); MNG-ENG arrangement (Fig. 2(b)). Specially, for thick cap layer the difference is obvious (see solid lines in Fig. 2). As it is seen from Fig. 2(a), we can have nearly omnidirectional dispersion curve in the ENG-MNG arrangement, whilst in the MNG-ENG arrangement, dispersion curve is limited to a narrow domain of incident angles for thick cap layer. More interestingly, one can see that the dispersion curves are intersected together at a single point on the \((\omega, \beta)\) plane for both the ENG-MNG and the MNG-ENG arrangements (point (1) in Fig. 2). In other words, there is a frequency and an angle of incidence for which, we can excite SWs independent of \(d_c\) and the type of the arrangement (ENG-MNG or MNG-ENG). The results of our investigations show that the structure with the set of material parameters chosen in Fig. 1 dose not support TM-polarized SMs. So, in what follows, we discuss about the properties of TE-polarized SMs.

To show the difference between SMs in two ENG-MNG and MNG-ENG arrangements we plotted the transverse structure of SMs versus distance \(z\) in Fig. 3. We see that the peak of the electric field is
Figure 3. (Color online) The transverse profile of the SWs vs coordinate $z$ for a) $d_c = 1d_1$, b) $d_c = 3d_1$, in the ENG-MNG arrangement and c) $d_c = 1d_1$, d) $d_c = 3d_1$ in the MNG-ENG arrangement corresponds to the point 1 in Fig. 2. Here $\beta = 1.5$, $\omega = 5$ GHz and the other parameters are same as Fig. 2.

appeared at the interface of the cap layer and the 1D-PC in the ENG-MNG arrangement, while in the case of MNG-ENG arrangement the peak of the electric field is located at the interface of the uniform medium and the cap layer. Here, the unusual structure of SMs comes from sharp jumps at the interface of layers resulting from opposite signs of permeability of adjacent layers and decaying evanescent waves in each layers.

By further inspection of dispersion curves we see that the dispersion curves have positive slopes. As the slope of the dispersion curve determines the corresponding group velocity of the mode and the direction of energy flow at the surface, we see that in our structure all modes are forward modes. On the other hand, Fredkin and Ron showed a combination of alternating layers with the ENG materials and layers with the MNG materials is equivalent to a LH medium [3]. Meanwhile, the structure containing LH medium can support backward modes. Although, backward wave propagation is the manifestation of left-handed electromagnetism [26] and not negative refraction which
1.5 2 2.5 3
0 1 2 3
\(d_c\)

Figure 4. (Color online) Existence regions for the SMs; the modes exist in the shaded regions for a) the ENG-MNG arrangement and b) the MNG-ENG arrangement. The other parameters are the same as the Fig. 2.

1.5 2 2.5 3
0 1 2 3
\(d_c\)

Figure 5. (Color online) Same as Fig. 2 for TM-polarized SWs with \(d_{ENG} = 0.8\) cm, \(d_{MNG} = 0.6\) cm and \(\frac{d_{ENG}}{d_{MNG}} > 1\).

occurs also at the interface of right-handed systems. In any case, the considered system does not support any backward modes.

The type of arrangement of the structure not only affects the dispersion curves of SMs, but also it changes the existence regions of TE-polarized SMs. To demonstrate this, we plotted the existence regions of the SMs at the second SNG band gaps on the parameter plane \((d_c, \beta)\) in Fig. 4 in which the shaded areas show the existence regions of the TE-polarized SMs. We find that the considered structure can support SMs for almost every \(\beta\) and \(d_c\) in the ENG-MNG arrangement. But, the existence region of SMs is restricted to a narrow domain of incident angles for relatively large \(d_c\) in the MNG-ENG arrangement.

Up to now, we discussed the property of TE-polarized SMs in the considered structure with \(\frac{d_{ENG}}{d_{MNG}} < 1\) which, it dose not allow TM-
Waves at the surface of a periodic multilayer structure

Figure 6. (Color online) Existence regions of TE and TM-polarized SMs for a) the ENG-MNG arrangement with the cap layer of the ENG material and b) the MNG-ENG arrangement with the cap layer of the MNG material as a function of the fraction of thickness of ENG layer to thickness of MNG layer. In the ENG-MNG arrangement $d_{\text{ENG}} = d_1$, $d_{\text{MNG}} = d_2$ and the ENG-MNG arrangement $d_{\text{ENG}} = d_2$, $d_{\text{MNG}} = d_1$. Here we used $\beta = 1.86$, $d_c = 0.01d_1$ (dotted lines), $d_c = 1d_1$ (dashed lines) and $d_c = 3d_1$ (solid lines). The unshaded regions show the first and the second band gaps of the periodic multilayer structure with SNG constituents, while the shaded regions indicate the corresponding pass bands.

polarized SMs. Our studies show that the structure can support TM-polarized SMs provided that $\frac{d_{\text{ENG}}}{d_{\text{MNG}}} > 1$. So, the occurrence of the TE-polarized and the TM-polarized SMs depends on the ratio of the thicknesses of the ENG layer to the MNG layer. To show the property of TM-polarized SMs we considered the structure with $d_{\text{ENG}} = 0.8\, \text{cm}$, $d_{\text{MNG}} = 0.6\, \text{cm}$. In Fig. 5 we plotted the dispersion curves of TM-polarized SMs on the plane of $\omega$ versus the propagation constant $\beta$ for different thickness of the cap layer ($d_c$). Same as TE-polarized SMs, the dispersion curves of TM-polarized SMs have positive slope. So, in our structure all modes are forward modes. To summarize the dispersion properties of polarized SMs, we plotted the existence regions of the TE and the TM-polarized SMs as a function of relative thickness of the ENG and the MNG layers ($\frac{d_{\text{ENG}}}{d_{\text{MNG}}}$). As one can see from the Fig. 6, the TE-polarized modes exist only for the relative thickness less than one and the TM-polarized modes exist only for the relative thickness greater than one in both the ENG-MNG (Fig. 6a) and the MNG-ENG (Fig. 6b) structures.

It must be mentioned that the dispersion relations of the surface polaritons of a slab made of a material which has dispersive permittivity and permeability, and is left-handed over a frequency
band in the microwave range of several GHz, has been investigated by Ruppin [27]. The author showed that his system can only support TM-polarized surface plasmon polariton (SPP) in the SNG frequency range which, this system is correspond to our ENG-MNG system without the periodic multilayer structure. On the other hand, the MNG-ENG system without the periodic multilayer structure is correspond to a MNG slab and one would expect SPPs for TE polarization. However, we see from Fig. 6 that in the presence of periodic multilayer structure it can be found both kind of TE and TM polarized SPP depending on the composition of the multilayer medium and the ratio of the thicknesses of the ENG layer to the MNG layer. This means, the existence or not of plasmons on the cap layer in the presence of the multilayer medium is very different from the case if the cap layer alone.

4. CONCLUSION

Briefly, we studied the linear SWs at the interface between an uniform RH medium and a semi-infinite 1D-PC made of SNG materials. We showed that, the considered structure can support only the TE-polarized SMs. These SMs depending on the arrangement of the layers of PC (MNG-ENG or ENG-MNG) can have two different transverse structures of field at the second SNG band gap of the PC. In the ENG-MNG arrangement the peak of SMs are located at the interface between the cap layer and the photonic crystal. But, in the MNG-ENG arrangement, the peak of SMs are located at the interface between uniform material and the cap layer. It is shown that in the considered ENG-MNG arrangement we can have nearly omnidirectional dispersion curves for thick cap layer. Moreover, we showed that there is a single point on the plane of $\omega - \beta$ for which the SM is independent of the thickness of the cap layer and the arrangement of the layers of PC. More interestingly, we demonstrated that the occurrence of the TE-polarized and the TM-polarized surface waves depends on the ratio of the thicknesses of the ENG layer to the MNG layer.

REFERENCES


