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Optical properties of graphene based annular photonic crystals

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1. Introduction
An annular Bragg reflector (ABR) as a periodic bilayer structure in an annular geometry plays an important and useful role in modern laser system with a feature of vertical emission (1). With the advance of modern fabrication techniques, ABR geometries become possible (2–5). Recently, annular Bragg resonators composed of ABRs surrounding a radial defect have been realized and demonstrated (6–9). The vertical emission feature of the annular Bragg resonators would make them of great use in optical communication and optoelectronics. In comparison with the conventional resonators, they also attract much interest because of their superior sensing ability in the biological and chemical applications (10). Optical properties of annular photonic crystal (APC) structures have been studied by many researchers and they reported very remarkable and interesting results in their studies (11–19).

Recently, the discovery of graphene (20) as a two-dimensional monoatomic layer of carbon atoms arranged in a honeycomb lattice (21–24) opened a new area in the field of material science. Graphene is one of the simplest two-dimensional conducting materials with remarkable electronic properties which have been revealed by transport measurements (22–25). Among these properties, the absorption is relatively flat for a monolayer 0.34 nm thick from visible to infrared wavelengths, with value as high as 2.3% (26). These absorption properties make the graphene a good candidate to replace transparent electrodes, optical display materials (27–29), and saturable absorbers for laser mode-locking (30, 31). Moreover, the interest in the optical response of graphene is even further boosted by recent progress of terahertz (THz) radiation technology. The frequency of THz radiation lies in the boundary region between light and radio waves. Customarily, the region is defined as $f = 0.3 - 10$ THz, $\lambda = 1000 - 30$ µm. Potential applications of such THz technology are widespread, including military security, medical diagnosis, coherent imaging, material analysis, environmental protection, and space science. Development of new photonic components dynamically functioning over such THz frequencies is a subarea of major currently ongoing advanced research effort and is very crucially relying on the availability of new materials. Very recently, the stack of graphene layers has been proposed such as THz modulators and broadband polarizer (32, 33). One of the important features of graphene is that its conductivity could be tuned by varying the chemical potential of the graphene sheets via electrostatic biasing (34–37). In the experimental realization, this voltage may be provided by electrodes which are THz-transparent dc conductors, such as thin-doped InSb films (38, 39). This feature provides a robust optical property of the PC that incorporates graphene (40). In addition, the real part of the conductivity (determining the attenuation) of graphene is remarkably small compared with noble
The geometry of the considered APC with the structure $I(DG)^N S$ is illustrated in Figure 1. The structure consists of the inner core region $I$ with refractive index $n_0$ (starting medium) and starting radius $\rho_0$. The surrounding medium $S$ of the structure has the refractive index $n_f$. The cylindrical layers $D$ are assumed made of the lossless dielectric materials with the refractive index $n_d$, the inner radius $\rho_i$ and the outer radius $\rho_{i+1}$ ($i = 0, 1, 2, \ldots, N$). Here, $N$ is the period number and $d = \rho_i - \rho_{i-1}$ is the radial thickness of the $D$ layers. It is assumed that the graphene monolayers $G$ with the negligible thickness $0.344$ nm are embedded between the adjacent dielectric layers. Experimentally, the considered graphene based APC may be fabricated through the following procedure. Using the chemical vapor deposition (CVD) method, the high-quality graphene monolayer, is transferred onto an appropriate dielectric layer (44, 45). Then, with the rolling methods the graphene-dielectric layer is rolled around a plastic rod or a plastic tube called mandrel. The mandrel can either be left as a part of a preform, thus forming a solid core, or it can be later removed for the fabrication of hollow core APC. Although the rolling method results in a spiral rather than a cylindrical structure, numerical simulations show that optical performances of the resulting APC are virtually identical (46, 47).

The optical properties of the graphene monolayers are described with the surface conductivity $\sigma(\omega)$ which is governed by the Kubo formula (48) including the intra band and inter band transition contributions as $\sigma(\omega) = \sigma_{\text{intra}}(\omega) + \sigma_{\text{inter}}(\omega)$, where

$$
\sigma_{\text{intra}}(\omega) = \frac{e^2}{4h} \frac{i}{2\pi} \left\{ \frac{16k_B T}{h\omega} \ln \left[ 2 \cosh \frac{\mu_c}{2k_B T} \right] \right\}
$$

$$
\sigma_{\text{inter}}(\omega) = \frac{e^2}{4h} \left\{ \frac{1}{2} \frac{1 + \frac{1}{\pi} \arctan \left( \frac{\hbar \omega - 2\mu_c}{\hbar \omega + 2\mu_c} \right)}{\left( \hbar \omega - 2\mu_c \right)^2 + (2k_B T)^2} \right\}
$$

Here, $e$ is the charge of the electron, $T$ is the absolute temperature, $k_B$ is the Boltzmann constant and $\mu_c$ is the chemical potential which is determined by the electron concentration and can be controlled by applying a gate voltage. These expressions show that the inter band contribution plays the leading role around the absorption threshold, $\omega \approx 2\mu_c$, while the intra band contribution is important at relatively low frequencies, $\omega < \mu_c$.

The cylindrical wave is assumed to be diverging radiially from the axis of symmetry $\rho = 0$ and then impinges normally on the first circular interface at $\rho = \rho_0$. Our goal is to find the reflection coefficient at the first circular boundary $\rho = \rho_0$, or the transmission coefficient at the final interface $\rho = \rho_N$. All electromagnetic fields are assumed to have an $\exp(i\omega t)$ time dependence. For cylindrical Bragg wave there exist two possible modes.
or polarizations which are transverse electric (E) and transverse magnetic (H) polarizations. \( E_z, H_\varphi \) and \( H_\rho \), are the nonzero fields for E-mode and \( H_z \), \( E_\varphi \) and \( E_\rho \), are the nonzero fields for H-mode. \( E_z \) and \( H_z \) satisfy the following governing equation:

\[
\frac{\partial}{\partial \rho} \left( \rho \frac{\partial F_z}{\partial \rho} \right) - \rho^2 \frac{1}{\alpha^2} \frac{\partial H_\rho}{\partial \rho} + \frac{\partial}{\partial \varphi} \left( \frac{\partial F_z}{\partial \varphi} \right) + \omega^2 \varepsilon \mu^2 F_z = 0, 
\]

(2)

where \( F_z = E_z, \alpha = -\mu \) for E-mode and \( F_z = H_z, \alpha = \varepsilon \) for H-mode. Here, \( \varepsilon \) and \( \mu \) are the permittivity and permeability of the cylindrical media. Using the separation of variables method, one can obtain the solution of Equation (2) in terms of the Bessel–Neumann or the Hankel functions. Here, we consider the traveling waves solution of Equation (2) in terms of the Hankel functions:

\[
F_z(\rho, \varphi) = AH^{(2)}_m(\kappa \rho) + BH^{(1)}_m(\kappa \rho) \exp(i \omega t),
\]

(3)

where \( A \) and \( B \) are constants, \( H^{(1)}_m(\kappa \rho) \) and \( H^{(2)}_m(\kappa \rho) \) are the Hankel functions of the first and second kind, \( m \) is the azimuthal mode number and \( k = \omega \sqrt{\varepsilon / \mu} = n \kappa \) is the wave number of the medium with the refractive index \( n \).

By applying the source-free curl Maxwell’s equations, the azimuthal parts of the magnetic and electric fields \( H_\varphi \) and \( E_\varphi \) can be expressed as:

\[
G_\varphi(\rho, \varphi) = iP \left[ AH^{(2)}_m(\kappa \rho) + BH^{(1)}_m(\kappa \rho) \right] \exp(i \omega t). 
\]

(4)

Here, \( P = \frac{\varepsilon - n^2}{\mu} \) and the primed quantities are the derivatives with respect to their own argument. For the simplest source that a uniform current passing through a line, the electric and magnetic fields are expressed as the lowest order of the Hankel functions. Equations (3) and (4) enable us to construct a single layer matrix relating the \( F_z \) and \( G_\varphi \) fields at its two interfaces. The matrix for the arbitrary \( i \)th layer (with interfaces at \( \rho_i \) and \( \rho_j \)) is written as

\[
\begin{bmatrix}
F_z(\rho_i, \varphi) \\
G_\varphi(\rho_i, \varphi)
\end{bmatrix} = D(k_1 \rho_1) T_i(\rho_i, \rho_j) D^{-1}(k_j \rho_j) \begin{bmatrix}
F_z(\rho_j, \varphi) \\
G_\varphi(\rho_j, \varphi)
\end{bmatrix},
\]

(5)

Here,

\[
D(k_\rho \rho) = \begin{bmatrix}
\frac{i \rho H^{(2)}_m(\kappa \rho)}{H^{(1)}_m(\kappa \rho)} & 1 \\
\frac{i \rho H^{(1)}_m(\kappa \rho)}{H^{(2)}_m(\kappa \rho)} & 1
\end{bmatrix},
\]

(6)

\[
D(k_\rho \rho) = \begin{bmatrix}
\frac{i \rho H^{(2)}_m(\kappa \rho)}{H^{(1)}_m(\kappa \rho)} & 1 \\
\frac{i \rho H^{(1)}_m(\kappa \rho)}{H^{(2)}_m(\kappa \rho)} & 1
\end{bmatrix},
\]

(7)

and

\[
T_i(\rho_i, \rho_j) = \begin{bmatrix}
\frac{H^{(2)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)} & 0 \\
0 & \frac{H^{(2)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)}
\end{bmatrix}.
\]

(8)

Applying boundary conditions at an arbitrary interface \( \rho_i \), which include the continuity of the tangential component of the electric field (\( E_z \) for E-mode and \( E_\varphi \) for H-mode), and the discontinuity of the tangential component of the magnetic field (\( H_\rho \) for E-mode and \( H_z \) for H-mode) caused by the induced surface current in graphene leads to:

\[
\begin{bmatrix}
F_+^+(\rho_i, \varphi) \\
G_+^+(\rho_i, \varphi)
\end{bmatrix} = M_\sigma \begin{bmatrix}
F_-^-(\rho_i, \varphi) \\
G_-^-(\rho_i, \varphi)
\end{bmatrix},
\]

(9)

where, + and – symbols indicate the fields at the right and the left of the interface respectively.

Here, \( \sigma(\omega) \) is the absorption coefficient.

\[
R = \frac{\text{Re} \left( \frac{H^{(1)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)} \right)}{\text{Re} \left( \frac{H^{(2)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)} \right)} |t|^2, 
\]

(10)

\[
T = \frac{\text{Re} \left( \frac{i P H^{(2)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)} \right)}{\text{Re} \left( \frac{i P H^{(2)}_{m_1}(k \rho_1)}{H^{(1)}_{m_1}(k \rho_1)} \right)} \frac{\rho_N}{\rho_0} |t|^2. 
\]

(11)

Here, \( r = \frac{M_{ii}}{M_{11}} \) and \( t = \frac{1}{M_{11}} \) where \( M_{ij} (i, j = 1, 2) \) are the elements of the following transfer matrix:

\[
M = D^{-1}(k_\rho \rho_0) M_\sigma \sum_{j=0}^{N} (D(k_\rho \rho) T(\rho_j, \rho_{j+1}) D^{-1}(k_\rho \rho)) \times (k_\rho \rho_{j+1}) M_\sigma D(k_\rho \rho_N). 
\]

(12)

The absorbance, which is defined as the fraction of energy dissipation in the graphene monolayers, is given by \( A = 1 - R - T \).

3. Results and discussion

Let us now make some numerical analysis for the optical spectra based on the above-derived equations. We limit our calculations only to the far-IR and THz regions at the frequency range of \( f < 10 \) THz due to the low losses of the graphene monolayers in this regime. We shall consider a hollow core APC which is surrounded by air with \( n_0 = n_f = 1 \). The refractive index of \( D \) layers is taken to be
n_d = 1.58 with the thickness d = 4.74 \mu m. The starting radius is \rho_0 = 18.97 \mu m.

The calculated (a) transmittance, (b) reflectance and (c) absorbance of our considered graphene based APC for the E polarization (solid lines) and the H polarization (dashed lines) at the azimuthal number m = 0 are given in Figure 2. Here, the chemical potential of the graphene monolayers is taken to be \mu_c = 0.2 eV with the period number N = 12. From the figure we see that the considered graphene based APC contains a nearly lossless high-reflection band gap at the considered frequency range.

The properties of this so called graphene induced band gap (GIPBG) have been studied in the planar geometry in great detail (40). It can be seen from Figure 2 that both E-polarized and H-polarized optical spectra of the structure are fairly similar. So, in continue we study only the E polarization and consider the effect of different geometrical and optical parameters on the spectra of the system.

Here, we want to study the effect of increasing the number of graphene monolayers on the spectra of the
APC. The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC for the different radii of the inner core region $\rho_0 = 189.7 \text{ nm}$ (dashed lines), $\rho_0 = 18.97 \mu\text{m}$ (solid lines) and $\rho_0 = 1.897 \text{ mm}$ (dash-dotted lines). Here, $N = 12$, $d = 4.74 \mu\text{m}$, $m = 0$ and $\mu_c = 0.2 \text{ eV}$.

![Figure 6](image)

Figure 6. The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC for the different radii of the inner core region $\rho_0 = 189.7 \text{ nm}$ (dashed lines), $\rho_0 = 18.97 \mu\text{m}$ (solid lines) and $\rho_0 = 1.897 \text{ mm}$ (dash-dotted lines). Here, $N = 12$, $d = 4.74 \mu\text{m}$, $m = 0$ and $\mu_c = 0.2 \text{ eV}$.

The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC for the different refractive indices of the inner core region $n_0 = 1$ (solid lines), $n_0 = 1.58$ (dashed lines) and $n_0 = 2.34$ (dash-dotted lines). Here, $N = 12$, $\rho_0 = 18.97 \mu\text{m}$, $d = 4.74 \mu\text{m}$, $n_d = 1.58$, $n_f = 1$, $m = 0$ and $\mu_c = 0.2 \text{ eV}$.

![Figure 7](image)

Figure 7. The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC for the different refractive indices of the inner core region $n_0 = 1$ (solid lines), $n_0 = 1.58$ (dashed lines) and $n_0 = 2.34$ (dash-dotted lines). Here, $N = 12$, $\rho_0 = 18.97 \mu\text{m}$, $d = 4.74 \mu\text{m}$, $n_d = 1.58$, $n_f = 1$, $m = 0$ and $\mu_c = 0.2 \text{ eV}$.

Now, the effect of the azimuthal mode number $m$ on the spectra of the structure is considered. In Figure 4, (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC are plotted at the different azimuthal mode numbers $m = 0$ (solid lines), $m = 3$ (dashed lines) and $m = 6$ (dash-dotted lines) for the E-modes. The figure reveals that the frequency range of the high reflectance GIPBG is enlarged with the increase
of the azimuthal mode number \( m \). More interestingly, the cylindrical Bragg waves with the higher azimuthal mode number \( m \) experience the lower absorbance in the corresponding GIPBG.

The geometrical parameters as the thicknesses of the dielectric layers \( D \) or the radius of the inner core region may affect the spectra of the used structure. To experience this, the optical spectra of the graphene based APC are plotted in Figures 5 and 6 for different thicknesses of the dielectric layers \( D \) and different radii of the inner core region, respectively. From Figure 5 it is seen that the frequency range of the high reflectance GIPBG is reduced by increasing the width of the dielectric layers \( D \) for the fixed radius of the inner core region. At the same time, the absorbance of the structure increases in the band gap frequency interval (see Figure 5(c)). However, the optical spectra of the structure are not affected by the changing the radius of the inner core region for the fixed width of the dielectric layers as seen in Figure 6.

Next, we shall investigate the effect of the optical parameters of the graphene based APC on the GIPBG of the structure. In Figures 7–9 the optical spectra of the structure are plotted for different refractive indices of the inner core region \( N \), the surrounding medium \( S \) and the dielectric layers \( D \), respectively. In Figure 7 the E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC are plotted for the different refractive indices of the inner core region \( n_0 = 1 \) (solid lines), \( n_0 = 1.58 \) (dashed lines) and \( n_0 = 2.34 \) (dash-dotted lines). Similar spectra are plotted in Figure 8 for three different refractive indices of the surrounding medium \( n_f = 1 \) (solid lines), \( n_f = 1.58 \) (dashed lines) and \( n_f = 2.34 \) (dash-dotted lines). From Figures 7 and 8 it is clear that the variations in the refractive indices of the inner core region or the surrounding medium do not have a considerable effect on the frequency range of the GIPBG and the absorbance of the structure in this band gap. Nevertheless, Figure 9 reveals that the change in the refractive index of the dielectric layers \( D \) significantly affects the optical spectra of the system. In Figure 9 the optical spectra of the system are plotted for three different refractive indices of the dielectric layers \( n_d = 1 \) (solid lines), \( n_d = 1.58 \) (dashed lines) and \( n_d = 2.34 \) (dash-dotted lines). As depicted in Figure 9, by increasing the refractive index of the dielectric layers, the width of the GIPBG decreases with the slight reduction in the absorbance.

Now, we explore the effect of the chemical potential of the graphene monolayers on the spectra of the system. The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC are plotted in Figure 10 for three different chemical potentials of the graphene monolayers \( \mu_c = 0.1 \text{ eV} \) (solid lines), \( \mu_c = 0.3 \text{ eV} \) (dashed lines) and \( \mu_c = 0.5 \text{ eV} \) (dash-dotted lines). As illustrated in Figure 10, the increase in the chemical potential of the graphene monolayers enlarges the frequency range of the high reflectance GIPBG with the slight growth in the absorbance of the structure.

Since in the practical realization of the graphene based APC it is difficult to have an ideal structure without defect, we want to consider the effect of structural defects on the spectra of the system. To do this, the E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC are plotted in Figure 11 for the structure with the random radius \( \rho_i = \rho_0 + id(1 + \delta) \) (\( i = 1, 2, \ldots, 12 \)). Here, \(-0.4 \leq \delta \leq 0.4 \) is a random number. For comparison, the spectra of the defectless ideal structure are shown here with the solid circles. As it is clear from Figure 11 the structural defects have a negligible effect on the optical spectra of the graphene based APC structure.

Finally, it is interesting to investigate the intensity distribution of the E-modes at the given frequency. Figure 12 shows \( \frac{\rho_0 |E_0(\rho, \phi)\|^2}{\rho_0 |E_o(\rho, \phi)\|^2} \) in the plane \(( \frac{x}{\rho_0}, \frac{y}{\rho_0} )\). Here, we considered two frequencies (a) 2 THz (inside the GIPBG) and (b) 3.3 THz (the edge of the GIPBG) correspond to the points 1 and 2 in Figure 10. The blue circles show the inner and the outer interfaces of the graphene based APC. The numerical calculations indicate that more than 99.84% of the incident intensity at the frequency 2 THz stays in the graphene based APC at the radial region \( 0 < \rho \leq \rho_N \) which 87.96% of the stored energy is confined within the hallow core region \( 0 < \rho \leq \rho_0 \).
Figure 11. The E-polarized (a) transmittance, (b) reflectance and (c) absorbance of the graphene based APC for the structure with the random radius $\rho_i = \rho_0 + \delta$ ($i = 1, 2, \ldots, 12$) correspond to Figure 12(d). Here, $-0.4 \leq \delta \leq 0.4$ is a random number. The other parameters are $N = 12$, $\rho_0 = 18.97 \mu m$, $d = 4.74 \mu m$, $n_0 = 1$, $n_f = 1$, $m = 0$ and $\mu_c = 0.2 \text{ eV}$.

Figure 12. The top view of the normalized intensity distribution $\frac{\rho |E_x(\rho, \phi)|^2}{\rho_0 |E_x(\rho_0, \phi)|^2}$ in the plane $(\frac{x}{\rho_0}, \frac{y}{\rho_0})$ at the frequencies (a) 2 THz and (b) 3.3 THz correspond to the points 1 and 2 in Figure 10.

4. Conclusion

By using the transfer matrix method in cylindrical Bragg waves, the optical properties of the graphene based annular photonic crystal have been numerically investigated. With the traveling field solutions of the cylindrical waves, we find that the structure contains a graphene induced photonic band gap. The band gap appears in the frequency below 10 THz which the graphene monolayers have low losses at this range. It is seen that this low loss high reflectance band gap is nearly independent from the polarization and remains almost invariant with the change in the period number, the radius of the inner core region and the refractive indices of the inner core region and the surrounding medium. We have shown that by increasing the azimuthal mode number and the chemical potential of the graphene monolayers the width of the graphene induced high reflectance band gap increases. However, the width of the band gap decreases by increasing the refractive index and the thickness of the dielectric layers.

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