Optical properties of one-dimensional photonic crystals containing graphene-based hyperbolic metamaterials

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A R T I C L E   I N F O

Article history:
Received 24 March 2016
Received in revised form 23 January 2017
Accepted 23 January 2017

Keywords:
Photonic crystal
Graphene monolayer
Metamaterial
Hyperbolic dispersion

A B S T R A C T

The transmission properties of a one-dimensional photonic crystal made of alternate layers of an isotropic ordinary dielectric and a graphene-based hyperbolic metamaterial are studied theoretically using the transfer matrix method. The metamaterial layers show hyperbolic dispersion in certain frequency range and are considered as an anisotropic effective medium in which the optical axis is normal to the graphene layers. It is shown that the structure has some photonic band gaps in both the hyperbolic and elliptical frequency regions of the hyperbolic metamaterial layers, which are tunable by changing the chemical potential of the graphene monolayers. Moreover, the characteristics of the transverse-magnetic (TM)-polarized photonic band gaps remarkably depend on the orientation of the optical axis of the hyperbolic metamaterial layers. It is found that the electric field intensity of the propagating modes from the hyperbolic metamaterial frequency region is concentrated in the high-index isotropic layers and the electric field intensity of the propagating modes from the elliptical frequency region is concentrated in the low-index anisotropic layers.

1. Introduction

Metamaterials are artificial subwavelength-structured media that exhibit unusual optical properties. One important kind of metamaterials that exhibit simultaneously negative permittivity ($\varepsilon$) and permeability ($\mu$) in a special frequency range are called double-negative (DNG) or left-handed (LH) materials. The other kind of metamaterials are single-negative (SNG) materials in which only one of the material optical parameters has negative value. A theoretical study of LH materials was made in 1967 by Veselago [1]. However, the experimental realization of such materials was demonstrated by Smith et al. in 2000 [2]. Metamaterials have attracted considerable attention because of their important potential to manipulate electromagnetic waves and control the propagation of light. Negative index of refraction, invisibility cloaks, perfect lenses, and hyperlenses that enable subwavelength far-field resolution are some potential applications of metamaterials [3–8].

Among the varieties of metamaterials, recently, hyperbolic metamaterials (HMMs) have rapidly gained a central role in the above-mentioned applications [9–12]. HMMs are a kind of anisotropic metamaterials in which dielectric permittivities in orthogonal directions have different signs [13,14]. In contrast to all commonly known media, in which iso-frequency dispersion surfaces are spheroids or ellipsoids, iso-frequency dispersion surfaces in HMMs are hyperboloids, determining the name hyperbolic. A few natural materials, including bismuth and graphite, exhibit hyperbolic properties in certain spectral ranges [15]. In the recent years, layered metal–dielectric structures and metallic nanowire arrays with hyperbolic dispersion have been experimentally realized across the optical spectrum [16–18].

One of the most important disadvantages of such structures is the lack of tunability and switching properties. Besides, high dissipative loss of HMMs because of the presence of the metal layers is another problem of these structures. To overcome these problems, a new kind of HMMs was proposed using graphene–dielectric multilayers [19–23]. Graphene, an allotrope of carbon atoms arranged in a one atom thick honey-comb lattice, has unique properties, such as high mobility of carriers, flexibility, robustness, and environmental stability [24,25]. In addition, at the frequency ranges of terahertz (THz) and far-IR, the dissipative losses of graphene are less than the usual metals and its electronic and optical responses are described by the surface conductivity, which is related to its chemical potential and can be controlled and tuned by voltage [26,27]. These unique characteristics have motivated scientists to study the graphene-based HMMs and demonstrated that such HMMs can be used in negative refraction [9], tunable broadband
hyperlens [22], tunable infrared plasmonic devices [28], and so on. More recently, the experimental realization of mid-infrared graphene-based HMMs has been demonstrated by Chang et al. [29].

Photonic crystals (PCs), which are artificially fabricated materials with periodic modulation in dielectric properties, have gained much attention because of their ability to create a range of forbidden frequencies known as photonic band gap (PBG). Thus far, the optical properties of the PCs containing various kinds of materials, including dielectrics, metals, semiconductors, and magnetic materials have been investigated [30–34]. A few years after the invention of metamaterials, the interest of some researchers has been directed toward the study of one-dimensional PCs (1D PCs) composed of LH materials [35]. It is shown that this kind of PCs show omnidirectional band gap, wherein their central frequencies and widths are invariant upon the change of scaling and insensitive to disorders, incident angles, and polarizations.

Furthermore, the optical properties and photonic band structure of 1D PCs containing graphene monolayers have been investigated more recently [36–39]. The authors in these works revealed that in the presence of the graphene sheets a new type of PBG is created in the THz region which is omnidirectional, insensitive to the polarization and tunable by a gate voltage. In 2014, Narimanov introduced photonic hypercrystals which include HMMs as constituent elements and showed that surface waves supported by a hypercrystal possess the properties of both the optical Tamm states in photonic crystals and surface-plasmon polaritons at the metal–dielectric interface [40]. However, to the best of our knowledge, the photonic band structure of 1D PCs containing graphene-based HMMs have not been discussed extensively in the literature.

Therefore, in the present paper, we are interested to study the transmission properties of a 1D PC made of alternating layers of an isotropic ordinary medium and a graphene-based HMM. We employ the transfer matrix method [41] to analyze the PBGs of the 1D PC for both the transverse-electric (TE) and TM polarizations. As mentioned before, this kind of metamaterials are uniaxial; hence, the effect of the orientation of the optical axis of the HMM layers on the transmission of the structure is investigated. It is found that the TM-polarized waves are strongly sensitive to the orientation of the optical axis while the TE-polarized waves are insensitive to its alignment. The tunability of the PBGs is also investigated by analyzing the influence of the chemical potential of the graphene sheets on the photonic band structure of the 1D PC. Finally, we represent the electric field distribution of the waves at some critical frequencies to show the behavior of the electromagnetic waves inside the 1D PC.

2. Theoretical model and method

We consider a 1D PC with the periodic structure \((AB)^{N}\) embedded in air, as shown in Fig. 1. Here, \(A\) represents an isotropic nonmagnetic dielectric layer with the permittivity \(\varepsilon_A = \varepsilon_0\) and the thickness \(d_A = 10 \mu m\), \(B\) is a uniaxial graphene-based HMM with thickness \(d_B = 10 \mu m\), and \(N = 8\) is the period number. Here, it is assumed that a plane wave is incident at an angle \(\theta\) upon the 1D PC from air and the layers are chosen to be parallel to the \((x-y)\) plane with the \(z\) axis normal to the interfaces of the layers.

The designed graphene-based B layer is shown in the inset of Fig. 1, consisting of graphene–dielectric multilayers. The thicknesses of the graphene monolayers is assumed to be \(t_g = 0.34\) nm and a dielectric with the permittivity of \(\varepsilon_d = 2.25\) and thickness \(t_d = 0.5\) \(\mu\)m is filling the space between the monolayers. If the graphene sheets are considered to be parallel to the \((x-y)\) plane, the relative permittivity of the graphene can be shown by \(\varepsilon_g = \varepsilon_{Gx} + i\varepsilon_{Gy}\), wherein \(\varepsilon_{Gx} = \varepsilon_{Gy} = \varepsilon_0\) is the permittivity in the plane of the graphene sheet and \(\varepsilon_g = 1\) is the out-of-plane permittivity of the graphene. Here, \(k_0 = \omega/c\) is the vacuum wavevector and \(\eta_0 = 377\Omega\) is the impedance of air [42]. \(\sigma_f\) is the surface conductivity of the graphene which is obtained from the Kubo formula [26], including the intraband and interband transition contributions as \(\sigma_f(\omega) = \sigma_f^{\text{intr}}(\omega) + \sigma_f^{\text{inter}}(\omega)\), where

\[
\sigma_f^{\text{intr}}(\omega) = \frac{ie^2}{4\pi\hbar} \ln \left( \frac{2|\mu_c| - (\omega_0 + i\omega / \Gamma)}{2|\mu_c| + (\omega_0 + i\omega / \Gamma)} \right),
\]

\[
\sigma_f^{\text{inter}}(\omega) = \frac{ie^2}{4\pi\hbar} \ln \left( \frac{2|\mu_c| - (\omega_0 + i\omega / \Gamma)}{2|\mu_c| + (\omega_0 + i\omega / \Gamma)} \right).
\]

Here, \(\parallel\) and \(\perp\) denote the directions parallel and perpendicular to the optical axis, respectively. When the graphene sheets are tilted relative to the \(z\) axis (the inset of Fig. 1), the optical axis is parallel to the \((x-z)\) plane and makes an angle \(\psi\) with the \(z\) axis; thus, the permittivity tensor of the HMM is given by

\[
\varepsilon_{\parallel} = \begin{pmatrix}
\varepsilon_{xx} & 0 & \varepsilon_{xz} \\
0 & \varepsilon_{\perp} & 0 \\
\varepsilon_{zx} & 0 & \varepsilon_{zz}
\end{pmatrix},
\]

where, \(\varepsilon_{xx} = \varepsilon_0\cos^2\psi + \varepsilon_1\sin^2\psi, \varepsilon_{zz} = \varepsilon_0\sin^2\psi + \varepsilon_1\cos^2\psi \) and \(\varepsilon_{xz} = \varepsilon_{zx} = (\varepsilon_1 - \varepsilon_0)\sin\psi\cos\psi\). For the TM-polarized waves, the magnetic field \(H = H_y(z)e^{i(kx-\omega t)}\), inside the HMM medium satisfies the wave equation

\[
\frac{d^2H_y}{dz^2} + \left( \frac{k_0^2 \varepsilon_{zz} - \varepsilon_{xx} k_0^2}{\varepsilon_{xz}} \right) H_y = 0,
\]

where \(k_0 = \omega\varepsilon_0\) is the \(x\) component of the wave vector. By applying the continuity condition to \(H_y\) and \(E_x\) at the interfaces and introducing a wavefunction as

\[
\psi(z) = \begin{pmatrix}
H_y \\
\omega k_0 E_x
\end{pmatrix},
\]

the following relationship is derived between the electric and magnetic fields at any two positions \(z\) and \(z + \Delta z\) of the same medium,

\[
\psi(z) = M_0(\Delta z, \omega)\psi(z + \Delta z).
\]
Here, $M_B$ is the transfer matrix of the HMM medium,

$$M_B(\Delta z, \omega) = e^{i\alpha_1 \Delta z} \begin{pmatrix} \cos(\alpha_2 \Delta z) & -i q_B \sin(\alpha_2 \Delta z) \\ -i q_A \sin(\alpha_2 \Delta z) & \cos(\alpha_2 \Delta z) \end{pmatrix},$$

(7)

where $\alpha_1 = (\varepsilon_{xz}/\varepsilon_{zz}) k_0$, $\alpha_2 = k_0 \sqrt{\varepsilon_{zz} - \sin^2 \theta}/\varepsilon_{zz}$ and $q_B = \varepsilon_{xz} \alpha_2 (\varepsilon_{zz} \alpha_1 k_0)$. Similar results can be obtained for the isotropic layer $A$,

$$M_A(\Delta z, \omega) = \begin{pmatrix} \cos(k_A^2 \Delta z) & -i q_A \sin(k_A^2 \Delta z) \\ -i q_A \sin(k_A^2 \Delta z) & \cos(k_A^2 \Delta z) \end{pmatrix},$$

(8)

where $k_A^2 = k_0 \sqrt{\varepsilon_A - \sin^2 \theta}$ is the $z$ component of the wavevector in the medium $A$ and $q_A = (1/\varepsilon_A) \sqrt{\varepsilon_A - \sin^2 \theta}$.

We use the well-known transfer matrix method to obtain the transmission of the structure as

$$t(\omega) = \frac{q_0 x_{11}(\omega) + q_1 x_{22}(\omega)}{q_0 x_{11}(\omega) + q_1 x_{22}(\omega) + q_2 q_0 x_{12}(\omega) + x_{21}(\omega)},$$

(9)

where $x_{ij}(\omega)$ are the elements of the total transfer matrix, $X(\omega) = [M_A(d_A)M_B(d_B)]^N$, and $q_0 = q = \cos \theta$. The results for the TE-polarized waves can be obtained similarly using the same procedure.

It should be mentioned here that effective medium theory is strictly valid for infinite periodic media and treating the graphene-based multilayer with finite periods by this formula is only an approximation for simplicity of the analysis. However, we will repeat our analysis for the case of $\varphi = 0$ using the transfer matrix method by considering the HMM layer as discrete graphene and dielectric layers and show that for the case of $\varphi = 0$, the approximation is rather good. For the case of $\varphi \neq 0$ the transfer matrix method is not a suitable approach and we can’t apply the boundary conditions for the inclined graphene monolayers.

### 3. Optical properties of the 1D PC: effective medium approach

In this section we study the optical properties of the structure using effective medium theory. Because of the frequency dependent surface conductivity of the graphene, its relative permittivity varies by the frequency. As a result, the elements of the permittivity tensor of the graphene-based HMM ($\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$) strongly depend on the frequency. To show this, we first assume that $\sigma_{\parallel}$ is given by Eq. (1) with $\mu_c = 0.25 \text{ eV}$, $\Gamma = 0$, and $T = 300 \text{ K}$ and plot the real and imaginary parts of $\varepsilon_{\parallel}$ and $\varepsilon_{\perp}$ as functions of frequency, as given in Fig. 2.

As it is clear from the figure, $\varepsilon_{\perp}$ is almost independent of the frequency and real value (i.e. $\text{Re}(\varepsilon_{\perp}) \approx \varepsilon_{\parallel} = \varepsilon_{\parallel}$). On the other hand, $\varepsilon_{\parallel}$ strongly depends on the frequency. According to Fig. 2(a) for the given parameters, $\text{Re}(\varepsilon_{\parallel})$ is negative in the frequency region below the critical frequency $f_c = 8.64 \text{ THz}$, which is shifted toward the higher frequencies by increasing the chemical potential $\mu_c$ of the graphene monolayers. The imaginary part of the $\varepsilon_{\parallel}$ is also frequency dependent at the hyperbolic dispersion region and reaches zero at the elliptical region.

Now, we limit our calculations only to the frequency range of 0–15 THz and investigate the effect of different orientations of the optical axis of the HMM layers on the transmission of the used structure. In Fig. 3, the transmission spectra of the system are represented as functions of $\theta$ and $f$ for two different values of (a) $\varphi = 0$ and (b) $\varphi = 30^\circ$. Here, the dark areas denote the band gaps and the bright areas show the allowed bands for the TE and TM polarizations. As observed, one cannot define a unique plane of incidence for normal incident waves in both the cases of isotropic and anisotropic
Fig. 4. The transmission of the 1D PC as functions of $f$ and $\varphi$ for the normal incidence of the waves. Here, we choose $\mu_c = 0.25$ eV and $T = 300$ K.

Fig. 5. The transmission of the 1D PC as functions of $f$ and $\mu_c$ for the normal incidence of the waves and $\varphi = 30^\circ$.

Materials. Thus, in contrast to the case of $\varphi \neq 0$, the transmission of the structure is identical for both TE and TM polarizations at the normal incidence (see Fig. 3). Moreover, the transmission spectra of the structure show the PBGs in both hyperbolic and elliptical frequency ranges of the HMM. However, unlike the TE polarization case, the transmission of the structure for the TM polarization strongly depends on the orientations of the optical axis of the HMM layers. Specifically, the transmission of the structure in the hyperbolic regime ($f < 8.64$ THz) is more sensitive to the orientations of the optical axis, $\varphi$.

To show the influence of $\varphi$ on the optical properties of the structure, the transmission spectra of the structure are plotted as functions of $f$ and $\varphi$ for both polarizations in Fig. 4. Here, we considered the case of normal incidence with $\mu_c = 0.25$ eV and $T = 300$ K. As noted earlier, the TE-polarized transmission spectra of the structure are independent of the orientation of the optical axis of the HMM layers while, we observe that the width of TM-polarized PBGs in the hyperbolic and the elliptical frequency ranges of the HMM layers strongly decreases by increasing $\varphi$.

As it is well known, the main advantage of the graphene-based structure is the tunability of its optical properties because of the controllable electronic and optical characteristics of the graphene. In this regard, $\sigma_\varphi$ is the important property of the graphene which is controlled by tuning the chemical potential $\mu_c$ by a gate voltage. We want to investigate the effect of the controllability of the surface conductivity of the graphene on the transmission properties of the used structure. To do this, the transmission spectra of the structure are plotted as functions of $f$ and $\mu_c$ for both TE and TM polarizations in Fig. 5. Here, we considered the case of normal incidence with $\varphi = 30^\circ$. It is observed that the width of the forbidden band gaps increases and the width of the allowed bands decreases by increasing $\mu_c$ in the hyperbolic and the elliptical frequency ranges of the HMM layers.

The transmission properties of the structure may be better understood by looking at the field profiles of the electromagnetic waves inside the structure at different frequencies. In Fig. 6, the spatial distributions of the tangential component of the electric field for the case of TE-polarized ($E_\varphi$, the solid lines) and TM-polarized ($E_\chi$, the dotted lines) normal incident waves are plotted inside the structure for $\varphi = 30^\circ$. Here, we considered the frequencies from (a) the middle of the second PBG, (b) the upper edge of the second PBG, (c) the middle of the third PBG, and (d) the upper edge of the third PBG. The other parameters are $\varphi = 30^\circ$, $\mu_c = 0.25$ eV, and $T = 300$ K.
PBG with \( \mu_c = 0.25 \) eV and \( T = 300 \) K. It must be noted that for the chosen parameters, the second PBG of the structure is located at the HMM region and the third one is completely at the elliptical dispersion regime.

Unlike the transmission property of the structure which is different for the TE and TM polarizations, Fig. 6 reveals that the behavior of the tangential electric field inside the structure is the same for both polarizations. Moreover, Fig. 6(a) and (c) shows that the tangential electric field intensity of the modes from the second PBG (the HMM frequency region) decays more rapidly in comparison to the modes from the elliptical region (the third PBG). On the other hand, the tangential electric field intensity of the mode from the upper edge of the second PBG has its maximum values in the isotropic high-\( \varepsilon \) A-layers (see Fig. 6(b)). From Fig. 6(d), we observe that the maximum values of the electric field intensity of the mode from the third PBG are located in the anisotropic B-layers. This different characteristic originates from the difference of the hyperbolic and elliptical frequency regimes. In the conventional PCs composed of the electromagnetic materials, the modes from the upper edge of the PBGs have their maximum values of the electric field intensity in the low-\( \varepsilon \) regions [45]. In our structure, the behavior of the modes from the third PBG in the elliptical region (Fig. 6(d)) is the same as the ordinary PCs, while the situation is inverse for the modes from the second PBG in the hyperbolic region (Fig. 6(b)).

To have a deeper insight in the transmission property of the used structure, we performed a numerical simulation that represents the intensity distribution \((|E|^2)\) of the TM-polarized Gaussian beams corresponding to the TM modes in Fig. 6 inside the periodic structure (see Fig. 7). Here, we considered the normal incident case with \( \psi = 30^\circ \). The width of the Gaussian beams are taken as \( a = 15\lambda \), where \( \lambda = c/f \) is the vacuum wavelength of the beams and the frequency of the beams are chosen as (a) \( f = 5.80 \) THz, (b) \( f = 8.30 \) THz, (c) \( f = 9.75 \) THz and (d) \( f = 11.18 \) THz. The simulations shown in Fig. 7 confirm that the decay length of the modes from the PBG of the HMM region is less than the ones from the PBG of the elliptical region. In addition, it is observed that the propagating modes from the HMM region concentrate their electric field intensity in high-index isotropic regions and the propagating modes from the elliptical region concentrate their intensity in low-index anisotropic regions.

4. Optical properties of the 1D PC: transfer matrix approach

As mentioned before, effective medium theory and mixing formula are only approximations for simplicity of the analysis. Because they do not consider the discrete nature of the whole structure, including the graphene-based multilayers. To investigate the accuracy of the effective medium theory for finite multilayers, in this section, we repeat our analysis for the case of \( \psi = 0 \) using the transfer matrix method by considering the HMM layers as discrete graphene and dielectric layers. In addition, we study the effect of losses; so, we consider the structure with scattering rate, \( \Gamma = 1 \) meV, which is a more realistic representation of available graphene technologies.

In Fig. 8(a), the transmission spectra of the 1D PC are represented as functions of \( \theta \) and \( f \) for \( \psi = 0 \). Here, the dark areas denote the band gaps and the bright areas show the allowed bands for the TE and TM polarizations. By comparing Fig. 8(a) with Fig. 3(a), it is observed that the similarity of the PBGs of the 1D PC is rather good in both approaches; however, the transmission values for the pass-bands are much lower because of the scattering losses which results to the disappearing of the first pass-band.

To investigate the effect of \( \mu_c \) on the transmission spectra of the 1D PC in the realistic approach, we have plotted the transmission spectra of the structure as functions of \( f \) and \( \mu_c \) for both TE and TM polarizations in Fig. 8(b). Here, we considered the case of normal incidence with \( \psi = 0 \). It is evident from the figure that the transmission of the structure strongly depends on the value of \( \mu_c \). Specially in the higher values of \( \mu_c \), the transmission of the 1D PC decreases remarkably and finally the pass-bands in the given frequency range are washed out.

Finally, we have studied the effect of the scattering losses of the graphene on the electromagnetic field profiles inside the structure. In Fig. 9 the spatial distributions of the tangential component of the electric field for the case of TE-polarized \( (E_y) \) normal incident waves are plotted inside the structure for \( \psi = 0 \). Here, we considered the frequencies from (a) the middle of the third PBG \( f = 9.70 \) THz and (b) the upper edge of the third PBG \( f = 10.92 \) THz with \( \mu_c = 0.25 \) eV and \( T = 300 \) K. Here, the solid lines and the dotted lines are obtained for \( \Gamma = 0 \) and \( \Gamma = 1 \) meV, respectively. It is observed that the effect of the scattering losses is not more considerable for the frequencies from the middle of the PBG; however, it has a more noticeable effect.
for the frequencies from the edge of the PBG because of the lower values of the transmission in the presence of the scattering losses.

5. Conclusion

In summary, the transmission properties of a 1D PC composed of alternate layers of an ordinary dielectric and a graphene-based HMM are investigated theoretically using the transfer matrix method. The HMM layers are considered as a homogeneous anisotropic medium with their optical axis normal to the graphene monolayers. It is observed that the structure has some PBGs in the both hyperbolic and elliptical frequency regions of the HMM layers. The results show that the orientation of the optical axis of the anisotropic medium has a remarkable effect only on the width and location of the TM-polarized PBGs. It is observed that the width of the PBGs strongly depends on the surface conductivity of the graphene and can be tuned by a gate voltage; thus, it can be used in the designing of the tunable far-IR filters and switches. Finally, we showed that the behavior of the electric field distribution of the modes from the HMM frequency region is completely different from the ones of the elliptical region.

References