Nonreciprocal optical isolation via graphene based photonic crystals

S. Roshan Entezar *, M. Karimi Habil

Physics Department, University of Tabriz, Tabriz, Iran

1. Introduction

Spatially nonreciprocal devices such as all-optical diodes and isolators are widely considered to be the key components for the next generation of all-optical signal processing. Replacing relatively slow electrons with photons as carriers of information would substantially increase the speed and the bandwidth of telecommunication systems, leading to a real revolution of the telecom industry. The optical nonreciprocity leads to different properties for opposite propagation directions of electromagnetic waves. A nonreciprocal response is usually related to the time-reversal symmetry breaking of the light-matter interaction [1]. One well-known example of the nonreciprocal response is the magneto-optical (MO) effect related to circularly polarized (CP) light propagation in gyrotropic materials. The biased magnetic field breaks the time-reversal symmetry, which leads to a nonreciprocal response of media [2,3]. At optical frequencies, typical MO materials (such as bismuth iron garnet [4,5]) exhibit a rather weak gyrotropic response. Nonetheless, significant response enhancements can be obtained via combining MO effects with the resonant effects of photonic crystals (PCs) [6–10]. PCs are periodic structures in one or more spatial directions. Such periodic structures exhibit a forbidden frequency region, called photonic band gap (PBG), in which electromagnetic waves can not propagate. PBG gives rise to unusual physical phenomena such as improving the efficiency of light sources [11,12], enhancing the electromagnetic energy absorption [13,14], improving the signal to noise ratio [15,16].

Recently, graphene has generated great interest among the scientific community due to its abundant potential applications in optoelectronic devices [17–22]. It consists of a two-dimensional mono-atomic layer of carbon atoms arranged in a honeycomb lattice. Graphene is one of the most simple and intriguing two-dimensional conducting material with unique optical and photonic properties. Among these properties, the absorption is relatively flat for a monolayer 0.34 nm thick from visible to infrared wavelengths, with values as high as 2.3% [23]. These absorption properties make graphene a good candidate to replace transparent electrodes, optical display materials [19,20,24], and saturable absorbers for laser mode-locking [25,26]. Moreover, the interest in the optical response of graphene is even further boosted by recent progress of terahertz (THz) radiation technology. The frequency of THz radiation lies in the boundary region between light and radio waves. Customarily, the region is defined as $f = 0.3 – 10$ THz, $\lambda = 1000 – 30$ $\mu$m. Potential applications of such THz technology are widespread, including military security, medical diagnosis, coherent imaging, material analysis, environmental protection, and space science. Development of new photonic components dynamically functioning over such THz frequencies is a subarea of major currently ongoing advanced research effort and is very crucially relying on the availability of new materials. Very recently, the stack of graphene layers has been proposed such as THz modulators and broadband polarizer [27,28]. Moreover, the graphene mono-layers have been proposed to obtain a giant Faraday rotation [29–31]. Such a giant Faraday rotation is due to the induced gyrotropic properties of the graphene mono-layers under the external magnetic field. In fact, in the presence of the static magnetic field, the electrons of the graphene would experience distinct
magnitudes of the Lorentz force from the right-handed CP (RCP) and the left-handed CP (LCP) waves. As a result, the propagation constant of the RCP and the LCP waves are different in the graphene mono-layer [29–31]. One of the important features of graphene is that its conductivity could be tuned by varying the chemical potential of the graphene sheets via electrostatic biasing [32–36]. This feature provides a robust optical properties of the PC that incorporates graphene [37]. In addition, the real part of the conductivity (determining the attenuation) of graphene is remarkably small compared with noble metal (e.g. Gold and Silver), which may possess desirable performance at THz frequencies.

In this paper, the transmission properties of the RCP and LCP waves in the graphene based PC are investigated using the well-known transfer matrix method at THz frequencies [38,39]. Since, the optical properties of the graphene mono-layer depends on the external magnetic field and the chemical potential. In Section 2, we present the model and the basic theory to investigate the transmission properties of the proposed structure. The general calculated results and the analysis of them are presented in Section 3. In Section 4, we summarize the obtained results.

2. PC structure and theoretical method

Consider a 1D PC created by alternating layers of the isotropic dielectric materials with the graphene mono-layers embedded between them (see Fig. 1). We assumed that the interface of the layers to be parallel to the x–y plane with the z-axis normal to the interface. The geometry of the PC structure as shown in Fig. 1 is (AGBG)N. Here, N is the period number of the PC structure. A and B are isotropic nonmagnetic dielectric layers with the thicknesses dA, dB, the relative permittivities εA, εB and the relative permeabilities μA = μB = 1, respectively. G is a graphene mono-layer which is treated as a homogenous film with the thickness dG = 0.344 nm [40,41] and the relative permittivity εG which is described based on the Kubo formula [30,32,33,42–44]

\[ 
\varepsilon_{eq} = 1 + i \frac{G_{intr} + G_{inter}}{\omega \mu_0 d_G} = \varepsilon_G - \frac{\omega_G^2}{\omega(\omega + jB)} 
\]

with \( i = \sqrt{-1} \) and \( \varepsilon_{inter} = 1 + i \frac{\omega_G}{\omega(\omega + jB)} \). Here, \( \omega_p \) is the plasma frequency, \( \omega \) is the angular frequency of the incident wave, \( \varepsilon_0, e, h, \) and \( k_0 \) are the electric permittivity of the vacuum, the electron charge, the reduced plank constant and Boltzmann constant, respectively. T is the temperature and \( \mu_e \) is the chemical potential. \( G_{intr} \) and \( G_{inter} \) are optical conductivity attributed to the intra-band and the inter-band transitions, respectively. The general expression for the conductivity of the graphene is obtained by Falkovsky and Varlamov [45]. Under the conditions \( h \omega_0, k_0 T \ll \mu_e \) as stated in the references [32,33]

\[ 
G_{intr} = \frac{\hbar \varepsilon^2 k_G T}{\pi \hbar^2 (\omega + j\gamma)} \left( \frac{\mu_e}{k_0 T} + 2 \ln(e^{-\mu_e/k_0 T} + 1) \right), 
\]

(2)

\[ 
G_{inter} = \frac{\varepsilon^2}{4h} \left[ \frac{1}{2} + \frac{1}{\pi} \tan^{-1} \left( \frac{\hbar \omega - 2 \mu_e}{2k_0 T} \right) - \frac{i}{2\pi} \ln \left( \frac{\hbar \omega + 2 \mu_e}{\hbar \omega - 2 \mu_e + (2k_0 T)^2} \right) \right]. 
\]

(3)

Here, \( \gamma \) is a damping parameter which shows the effect of the losses in the graphene. By applying a perpendicular external magnetic field \( B \) in the z direction (see Fig. 1), the graphene mono-layer shows gyrotropic properties. In such a case, the permittivity of the graphene mono-layer is described by the following tensor [30]

\[ 
\hat{\varepsilon} = \begin{pmatrix} 
\varepsilon & i \varepsilon_g & 0 \\
-i \varepsilon_g & \varepsilon & 0 \\
0 & 0 & \varepsilon_z 
\end{pmatrix}. 
\]

(4)

Here, \( \varepsilon = \varepsilon_{inter} - \frac{\omega_G^2}{\omega(\omega + jB)} \) and \( \varepsilon_g = \frac{\omega_G^2}{\omega(\omega + jB)} \). Here, \( \hat{\varepsilon} = \frac{\varepsilon x - i \varepsilon_y}{\sqrt{2}} \) one can diagonalize the tensor \( \varepsilon \) as \( \hat{\varepsilon} = R \hat{\varepsilon} \hat{R} \) [46]. Here, the rotation matrix \( R \) is given by

\[ 
R = \begin{pmatrix} 
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1 
\end{pmatrix}, 
\]

(5)

and

\[ 
\hat{\varepsilon} = \begin{pmatrix} 
\varepsilon_s & 0 & 0 \\
0 & \varepsilon_c & 0 \\
0 & 0 & \varepsilon_z 
\end{pmatrix}, 
\]

(6)

where,

\[ 
\varepsilon_s = \varepsilon_{inter} - \frac{\omega_G^2}{\omega(\omega \pm j\mu_e + jB)} \]. 

(7)

are the permittivity of the graphene mono-layer for the RCP (+) and the LCP (−) waves, respectively. Since, the permittivities of the isotropic layers A and B are identical for both CP waves, the dynamic equations of the CP waves in the new basis \( (\varepsilon_s, \varepsilon_c, \varepsilon_z) \) must be identical to the equations of the linearly polarized waves in the \( (\varepsilon_x, \varepsilon_y, \varepsilon_z) \) basis. Thus, one can use the \( 2 \times 2 \) transfer matrix method to obtain the spectra of the structure. As one know the electric and the magnetic fields of the CP waves at two different position z and \( z + \Delta \) inside a layer (say the ith layer) are connected via the following equation [38,39]:
where,

$$M^\pm_i(\Delta) = \begin{pmatrix} \cos(k^+_i \Delta) & (-i \frac{a}{c}) \sin(k^+_i \Delta) \\ (-i \frac{a}{c}) \sin(k^+_i \Delta) & \cos(k^+_i \Delta) \end{pmatrix},$$

$$I = (A, B \text{ or } G), k^+_x = (\omega/c)c_0^2, k^+_y = (\omega/c)c_0^2 \text{ and } k^+_z = (\omega/c)c_0^2 .$$

Using Eq. (8) the electric and the magnetic fields at the input ($z = 0$) and exit sides ($z = L$) of the structure are related via the following matrix equation:

$$\begin{pmatrix} E^x(z = L) \\ H^x(z = L) \end{pmatrix} = M^\pm \begin{pmatrix} E^x(z = 0) \\ H^x(z = 0) \end{pmatrix} ,$$

Here, $L = N(d_x + d_y + 2d_z)$ and $M = (M^x_i(d_x)M^y_i(d_y)M^z_i(d_z))^N$ is the total transfer matrix of the structure. Then, the transmission and the reflection of the structure for the normally incident CP waves are given by:

$$T^\pm(\omega) = \frac{2}{|M^x_{11} + M^x_{22} + M^x_{21} + M^x_{12}|} ,$$

$$R^\pm(\omega) = \frac{|M^x_{22} - M^x_{11}| - |M^x_{21} - M^x_{12}|}{|M^x_{11} + M^x_{22} + M^x_{21} + M^x_{12}|}.$$

The absorption, which is defined as the fraction of the energy dissipated in the graphene mono-layers, is given by $A^\pm(\omega) = 1 - R^\pm(\omega) - T^\pm(\omega)$ for both CP waves.

### 3. Results and numerical calculations

In what follows, we consider the geometrical and optical parameters of the structure to be $d_x = d_y = 10 \mu m, d_z = 5, \epsilon_g = 2.5$ and $N = 10$. As one know the chemical potential, the mobility and the damping parameter of the graphene mono-layer are temperature dependent parameters. Specifically, the chemical potential and the carrier mobility decrease by increasing the temperature. However, the density of the carriers and hence the chemical potential of the graphene mono-layer can be controlled by a gate voltage or molecule doping [32,47–49]. In this work we consider $\mu_c = 0.1 eV$ and $\gamma = 0.6 THz$ at the room temperature $T = 300 \ K$. These parameters are correspond to the experimentally obtained electron mobility $6 \times 10^5 - 2 \times 10^6 cm^2 / Vs$ [30,50]. With the considered parameters, the plasma frequency will be $\omega_p = 2000$ THz and by assuming $B = 1 T$ we get $\omega_L = 9 THz$ for the angular cyclotron frequency.

First of all, we want to investigate the dispersion of the permittivity tensor of the graphene mono-layer. To do this, the real (the solid lines) and the imaginary (the dotted lines) parts of a) $\epsilon$, b) $\epsilon_\|$, c) $\epsilon_\perp$, and d) $\epsilon_\perp$ are plotted as functions of the frequency $f$ in Fig. 2. As it is seen from the figure, the sign of $Re(\epsilon), Re(\epsilon_\|)$ and $Re(\epsilon_\perp)$ changes at the frequency $f < 1.4 THz$ which is the cyclotron frequency ($f_c = \omega_c/2\pi = 1.4 THz$). However, $Re(\epsilon_\perp)$ is negative at the considered frequency interval. Moreover, $Im(\epsilon)$ and $Im(\epsilon_\perp)$ are positive and $Im(\epsilon_\perp)$ is negative with their peak values at $f = 1.4 THz$. Nevertheless, $Im(\epsilon_\perp)$ is positive and monotonically decreases by increasing the frequency $f$.

Since, the propagation of the RCP wave in the graphene mono-layer is affected only by $\epsilon_\perp$, and the propagation of the LCP wave depends only to $\epsilon_\perp$, we want to study the optical spectra of the PC structure for both RCP and LCP waves. In Fig. 3 the transmission, reflection and absorption spectra of the PC structure are plotted as functions of the frequency $f$ at the normal incidence for the LCP (the left panels) and the RCP (the right panels) waves. Here, the solid lines show the spectra in the absence of the external magnetic field and the dotted lines show the situation in the presence of the external magnetic field. As it is clear from Fig. 3, the structure has two PBGs for both LCP and RCP waves at the similar frequency ranges in the absence of the external magnetic field. This is due to the isotropic behavior of the graphene mono-layer in the absence of the external magnetic field which causes the optical parameters of the graphene mono-layers to be the same for both CP waves (i.e. $\epsilon_\perp = \epsilon_\perp = \epsilon_{max} - \frac{\omega_p^2}{\omega^2}$. As stated in the reference [37], the low frequency PBG which is called the graphene induced photonic band gap (GIPBG) is due to the graphene lattice and the

![Fig. 2](image-url)
second band gap is the usual Bragg gap. Also, the absorption of the PC structure in the frequency of the band gaps is very low for both CP polarized waves. In the presence of the applied external magnetic field, the GIPBG for the LCP wave is blue shifted and the GIPBG for the RCP wave is red shifted (see the dotted lines in Fig. 3). Since, the Bragg gap of the structure is mainly due to the dielectric lattice of the structure, it is not considerably affected by the external magnetic field.

In order to further explore the effect of the external magnetic field on the PBGs of the structure, the optical spectra of the PC structure are plotted in Fig. 4. Here, the transmission spectra of the PC structure are shown in the plane of the external magnetic field $B$ and the frequency $f$ for a) the LCP and b) the RCP waves. Here, the dark and the white areas represent the gaps and the transmission bands of the structure, respectively. As it is clear from Fig. 4, the GIPBG of the PC structure for the LCP wave is blue shifted by increasing the applied external magnetic field without a considerable effect on the width of the GIPBG. On the other hand, the figure reveals that only the upper edge of the GIPBG of the PC structure for the RCP wave is red shifted by increasing the applied external magnetic field which considerably decreases the width of the GIPBG. Furthermore, we see that the Bragg gap of the structure
for the LCP (RCP) wave slightly blue (red) shifted without considerable change in its width which is due to the negligible effect of the graphene lattice on the Bragg gap of the PC structure.

Since, the chemical potential of the graphene mono-layers can be tuned via a gate voltage, it is interesting to examine the effect of the chemical potential on the transmission spectra of the PC structure. In the experimental realization, this gate voltage may be provided by electrodes which are THz transparent dc conductors [34–36,51]. In Fig. 5, the transmission spectra of the PC structure are plotted in the plane of \((\mu_c, f)\) for the LCP (the left panels) and the RCP (the right panels) waves. Here, three different values are considered for the external magnetic field. In Figs. 5(a,b) \(B = 0\), in Figs. 5(c, d) \(B = 1\) T and in Figs. 5(e, f) the considered magnetic field is applied in the opposite direction (i.e. \(-z\) direction), so \(B = -1\) T. As one can see from the Figs. 5(a,b), in the absence of the external magnetic field, the effect of increasing the chemical potential of the graphene mono-layers on the transmission spectra of the structure is identical for both CP waves. In the presence of 1 T magnetic field, the GIPBG of the structure for the LCP wave is blue shifted by decreasing the chemical potential \(\mu_c\) (see Fig. 5(c)). However, only the upper edge of the GIPBG of the structure for the RCP wave is red shifted by decreasing the chemical potential \(\mu_c\) (see Fig. 5(d)). By comparing Figs. 5(c, d), we see that there are some regions in which only one of the CP waves can be transmitted from the structure. The selected frequency lies in the propagating band of the structure. Here the solid lines show \(|E(z)|\) for both CP waves in the absence of the external magnetic field, the dotted lines show \(|E(z)|\) in the presence of the external magnetic field with \(B = 0.7\) T and the dashed lines indicate the case of \(B = -0.7\) T (i.e. the external magnetic field applied in \(-z\) direction).

In order to show that the PC structure can be used as an optical isolator, we examine the spatial distribution of the electric fields of both CP waves inside the PC structure. We can use Eq. (8) with

\[
\begin{bmatrix}
E^x(z = 0) \\
H^z(z = 0)
\end{bmatrix} = \begin{bmatrix}
1 + r^z \\
-(1 - r^z)
\end{bmatrix} E^o_z,
\]

\[E^{\pm}(z = 0) = \begin{bmatrix}
M_{11} - M_{21}M_{12}^{-1}M_{22} & -M_{11} + M_{21}M_{12}^{-1}M_{22} \\
M_{12}^{-1}M_{21} & M_{12}^{-1} - M_{21}
\end{bmatrix} E^o_z.
\]

\[(13)\]

to obtain the spatial distribution of the electric fields of the CP waves inside the structure. Here, \(r^z = \frac{M_{11} - M_{12}M_{21}^{-1}M_{22}}{M_{12}^{-1}M_{21} - M_{22}}\) is the reflection coefficient of the PC structure and \(E^o_z\) is the electric field of the incident CP wave. The spatial distributions of the electric field of the RCP (the upper panel) and the LCP (the lower panel) waves inside the PC structure are plotted in Fig. 6 at the frequency \(f = 1.5\) THz.

The selected frequency lies in the propagating band of the structure in the absence of the external magnetic field. Here, the solid lines show \(|E(z)|\) for both CP waves in the absence of the external magnetic field, the dotted lines show \(|E(z)|\) in the presence of the external magnetic field with \(B = 0.7\) T and the dashed lines indicate the case of \(B = -0.7\) T. It is seen that the electric field of both CP waves have oscillatory behaviors in the absence of the external magnetic field. This is the characteristic of a propagating wave which can be transmitted from the PC structure. By applying the external magnetic field in the \(z\) direction, the RCP wave still propagates through the structure and is transmitted from it. While, the electric field of the LCP wave decays inside the structure. So, it is reflected from the...
PC structure. For the magnetic field applied in the reverse direction, the RCP wave is reflected and the LCP wave is transmitted from the structure.

In Fig. 7 we considered the CP waves at the frequency $f = 0.89$ THz. This frequency is selected from the GIPB of the structure in the absence of the external magnetic field. As it is clear from the figure in the absence of the magnetic field, the electric field of both CP waves decay inside the structure. So, they cannot propagate in the structure and are reflected from it (see the solid lines in Fig. 7). However, by applying an appropriate magnetic field with the proper direction only one of the CP waves is transmitted from the structure and the other one is reflected from it (see the dotted and the dashed lines in Fig. 7). Instead of reversing the direction of the external magnetic field, we can imagine that the waves are incident from the exit side of the structure. This means that the structure can transmit only one of the CP waves in a given direction and it transmits the other CP wave in the opposite direction. At the same time, the transmitted CP wave is reflected from the structure in the opposite direction.

In the calculation presented in this paper we neglected the effect of the interface material on the dielectric function or optical conductivity of the graphene. Also the optical conductivity of the graphene mono-layer, the graphene based PC structure can become modified when it is interfaced with another material and may change slightly the transmission properties of the layered stack. However, it does not alter the main results obtained in the paper.

4. Conclusion

Using the transfer matrix method, the transmission properties of the graphene based PC structure have been studied in the presence of an externally applied magnetic field. Due to gyrotropic response of the graphene mono-layer, the graphene based PC structure may shows nonreciprocal properties. By applying an appropriate magnetic field, the structure can discriminate between the LCP and the RCP waves impinging to it. The results indicate that the PC structure can be used as a tunable optical isolator for preventing undesired backward reflection.

References


G.W. Hanson, Quasi-transverse electromagnetic modes supported by a graphene parallel-plate waveguide, J. Appl. Phys. 104 (2008) 084314.


J. Sakurai, Modern Quantum Mechanics, Addison-Wesley, 2011.


